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DYNAMICS AND ROBUST CONTROL OF A SAMPLED DATA SYSTEM FOR LARGE SPACE STRUCTURES

Volume 2: The LQG/LTR Methodology for the Discrete-time System and the Design of Reduced Order Robust Digital Controller for Orbiting Flexible Shallow Spherical Shell System

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The analysis and design of robust controllers for multivariable discrete-time feedback systems in the frequency domain are studied. The robustness stability conditions for discrete-time systems with additive alteration, and with multiplicative alteration are developed.

The robust LQG/LTR method has been extended from the continuous-time system to the discrete-time system. It has been proven that the LQG/LTR method is also valid for the LQG control of the discrete-time system with the filtering observer.

As an application of the LQG/LTR technique for discrete-time systems, the design of reduced order optimal digital LQG controllers for the orbiting flexible shallow spherical shell system is considered. The comparison between the digital optimal LQG controller with the filtering observer and with the predicting observer for the orbiting flexible shallow spherical shell has been made.

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SUMMARY

To develop the analysis and design methods for robust control of large space structure sampled data stochastic systems , the theory of multi–input, multi–output transfer function matrix with the z–transformation is used. The analysis and design of robust control of multivariable discrete–time feedback systems in the frequency domain are studied. The singular value theory is used to establish robustness stability criterian for discrete–time systems in the presence of parameter uncertainties or unmodelled dynamics in the frequency domain. The robustness stability conditions for discrete–time systems with additive alteration, and with multiplicative alteration are developed. The linear–quadratic–regulator(LQR) and linear–quadratic–Gaussian(LQG) theory are used for studying the loop transfer recovery(LTR) of discrete–time systems, and the robust LQG/LTR method has been extended from the continuous–time systems to the discrete–time systems. It has been proven that the LQG/LTR method is also valid for the LQG control of the discrete–time systems with the filtering observer.

As an application of the LQG/LTR technique for discrete–time systems, the design of reduced order optimal digital LQG controllers for the orbiting flexible shallow spherical shell system is considered.. Simulations have certified the 12–dim. reduced order controllers will be sufficient for the optimal LQG control of the orbiting shallow spherical shell system in the presence of unmodelled dynamics. The performance of the 8–dim. reduced order LQG controllers for the shell system is unacceptable. The 6–dim. reduced order controllers for the shell system will result in the severe divergence of the transient responses.

The comparisons between the digital optimal LQG controller with the filtering observer and predicting observer for the orbiting flexible shallow spherical shell system have been made. Based on a comparison of the robustness recovery properties, the system performance of the robust control system with the filtering observer has better transient response characteristics than the LQG robustness control system with the predicting observer.

PREFACE

This research symbolizes the work performed during the period September 1989 to August 1991 on WL Contract No. F33615–89C–3225. The first volume of this contract report is about the optimal linear quadratic regulator digital control of a free–free orbiting platform which is based on a master's thesis submitted to Howard University by one of the authors, Aprille Joy Ericsson. The second volume of this contract report is about the LQG/LTR methodology for discrete–time systems, and the analysis and design of robust digital LQG/LTR controllers for large flexible space structural systems

Special appreciation is extended to Dr. V.B. Venkayya, WL/FIBRA Principal Scientist and Project Engineer who directed this effort and helped provide support. Also the authors thank Mr. Duane Veley who was also the project engineer for this research. They provided helpful comments and constructive criticism.

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Introduction

The purpose of this research is to study and develop the analysis and design methods for robust control of the large space structure sampled data stochastic system with a specific application to the orbiting flexible shallow spherical shell system.

It is well known that one of the most important breakthroughs in multi-input, multi-out feedback system theory for the last decade is the development of the loop transfer recovery methodology for the linear quadratic Guanssian problem which is called LQG/LTR. Unfortunately, the previous research works in this field are almost all for the continuous-time system. Therefore, we must study and address the following problems before applying the LQG/LTR technique to design discrete-time system robust controllers for the orbiting large space structural system.

(1) How to get the frequency response of the transfer function matrix for the discrete-time systems supposing the discrete-time system model is given in the time domain ?
(2) What is the robustness stability condition for the discrete-time system in the frequency domain ?
(3) Is there loop transfer recovery for the discrete-time systems ? How to prove it ?

In this report, the first three Chapters will address the three problems. As an application of the LQG/LTR technique for the discrete-time system, Chapter 4 studies the design problem of the reduced order optimal digital LQG controller for the orbiting flexible shallow spherical shell system.

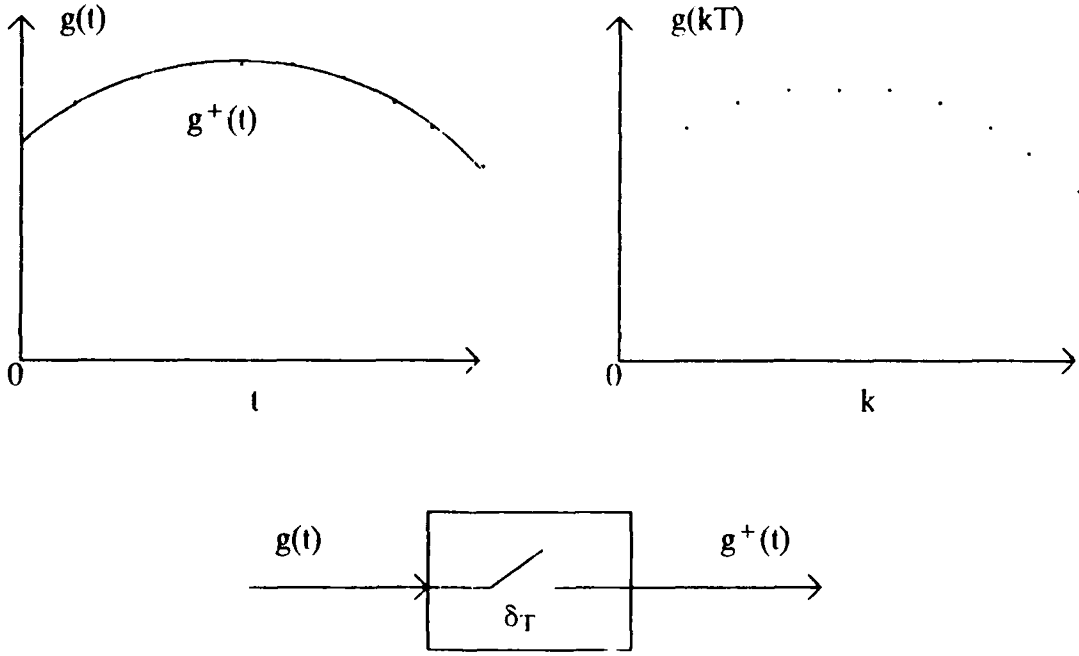
Chapter 6 considers the differences between the digital optimal LQG controller with filtering observer and predicting observer for the orbiting flexible shallow shell systen. The general conclusions and suggested future direction will be given in Chapter 7.

1 The Analysis Method in the Frequency Domain for Discrete-time Systems

As we know, a lot of research about robust control has been completed for continuous-time systems and the Laplace transformation method is its main analysis tool in the frequency domain. What we want to do is to find the relationships between the transfer functions of the continuous-time system and the discrete-time system in the frequency domain , so that the research results of the robustness problem for the continous-time system may be applied to the discrete-time system.

1.1 Discrete-time function and equivalent continous-time function

It is supposed that $g(t)$ is a continous-time function, $g(kT)$ is the sampling function of $g(t)$ at $t=kT$ points ($k=1,2,\ldots$), i.e. $g(kT)$ may be looked at as the output of an ideal impulse sampler for which the input is the continous-time function, $g(t)$.



Ideal Impulse Sampler

Fig. 1-1 The discrete-time function and its equivalent continous-time function

The output of the ideal impulse sampler can be written mathematically as follows:

$$g^+(t) = \sum_{k}^{\infty} g(t)\ \delta(t-kT) = \sum_{k}^{\infty} g(kT)\ \delta(t-kT) \qquad (1-1)$$

Then $g^+(t)$ can be called the equivalent continous-time function associated with the discrete-time function $g(kT)$ and its Laplace transformation is

$$G^+(s) = L\{ g^+(t) \} = \int_0^{\infty} \sum_k^{\infty} g(kT)\ \delta(t-kT)\ e^{-st}\, dt = \sum_k^{\infty} g(kT)\ e^{-kTs} \qquad (1-2)$$

The Z-transformation of g(kT) is

$$G(z) = Z\{ g(kT) \} = \sum_k^{\infty} g(kT)\ z^{-K} \qquad (1-3)$$

It is evident that the relationship between tne Laplace transformation of the equivalent continous-time function $g^+(t)$ associated with the discrete-time function g(kT), and the Z-transformation of the discrete-time function g(kT) is

$$G(z) = G^+(s)\big|_{s=(\ln z)/T} \quad \text{or} \quad G^+(s) = G(z)\big|_{z=e^{sT}} \qquad (1-4)$$

It will be proven that this relationship (1-4) will still be true for the transfer functions between the discrete-time system and the equivalent continous-time system.

1.2 The relationship between the input and output for discrete-time systems

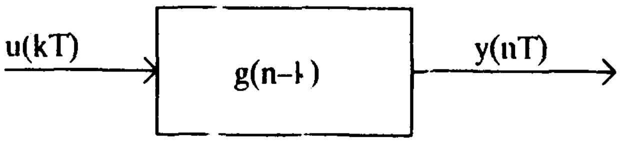


Fig. 1-2 The relationship between input and output for the discrete-time system

It is supposed that the input and output for the discrete-time systems is u(kT), and y(kT), respectively. The relationship between the input and output in the time domain is

$$y(nT) = \sum_k^{n-1} g((n-k)T)\ u(Tk) = \sum_k^{\infty} g((n-k)T)\ u(Tk) \qquad (1-5)$$

(because $g((n-k)T)=0$ when $k > n-1$)

where g(nT) is an impulse response function of the discrete-time system.

The Z-transformation of the output y(nT) is

$$Y(z) = \sum_{n}^{\infty} y(nT)\, z^{-n} = \sum_{n}^{\infty} \sum_{k}^{\infty} g\,((n-k)T)\; u(kT)\; z^{-n}$$

$$= \sum_{m}^{\infty} \sum_{k}^{\infty} g\,(mT)\; u(kT)\; z^{-(m+k)} \qquad (n-k=m)$$

$$= \sum_{m}^{\infty} g(mT)\, z^{-m} \sum_{k}^{\infty} u(kT)\, z^{-k} \qquad (g(mT)=0\ ,\ m<0\ )$$

$$= G(\,z\,)\quad U(z\,) \tag{1-6}$$

where

$$G(z) = \sum_{m}^{\infty} g(\,mT)\, z^{-m} = Z\{\, g(mT)\}$$

$$U(z) = \sum_{k}^{\infty} u(\,kT\,)\, z^{-k} = Z\{u(kT)\}$$

1.3 The relationship between the input and output of the equivalent continuous-time system

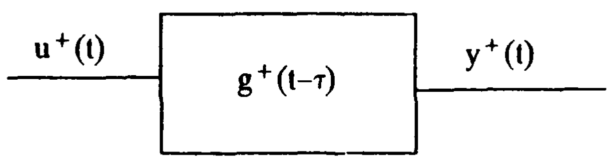


Fig. 1-3 The relationship between input and output for the equivalent continous-time sytem

It is assumed that the input and output for the equivalent continious-time system is $u^{+}(t)$, and $y^{+}(t)$, respectively.The relationship between the input and output in the time domain is:

$$y^{+}(\,t\,) = \int_{0}^{\infty} g^{+}(\,t-\tau\,)\; u^{+}(\,\tau\,)\; d\,\tau \tag{1-7}$$

where

$$u^{+}(\,t\,) = u(\,t\,)\; \delta_{T}(\,t\,) = \sum_{k}^{\infty} u(\,kT\,)\; \delta(\,t-kT)$$

$$g^{+}(\,t\,) = g(\,t\,)\; \delta_{T}(\,t\,) = \sum_{k}^{\infty} g(\,kT\,)\; \delta(\,t-kT)$$

$$y^+(t) = y(t)\,\delta_T(t) = \sum_k^{\infty} y(kT)\,\delta(t-kT)$$

$$\delta_T(t) = \sum_k^{\infty} \delta(t-kT) \qquad \text{the impulse sampler}$$

The Laplace transformation of (1–7) can be expressed as:

$$Y^+(s) = \int_0^{\infty} y^+(t)\, e^{-st}\, dt = \int_0^{\infty}\int_0^{\infty} g^+(t-\tau)\, u^+(\tau)\, e^{-st}\, dt\, d\tau$$

$$= \int_0^{\infty}\int_0^{\infty} g^+(\lambda)\, u^+(\tau)\, d\lambda\, e^{-s(\lambda+\tau)}\, d\tau \qquad (t-\tau=\lambda)$$

$$= \int_0^{\infty} g^+(\lambda)\, e^{-s\lambda}\, d\lambda \int_0^{\infty} u^+(\tau)\, e^{-s\tau}\, d\tau \qquad (g^+(\lambda)=0\ ,\ \lambda< 0)$$

$$= G^+(s)\ \ U^+(s) \tag{1-8}$$

where

$$G^+(s) = \int_0^{\infty} g^+(\lambda)\, e^{-s\lambda}\, d\lambda = L\{g^+(\lambda)\}$$

$$U^+(s) = \int_0^{\infty} U^+(\lambda)\, e^{-s\lambda}\, d\lambda = L\{u^+(\lambda)\}$$

$$Y^+(s) = \int_0^{\infty} y^+(\lambda)\, e^{-s\lambda}\, d\lambda = L\{y^+(\lambda)\}$$

Because

$$G^+(s) = \int_0^{\infty} g^+(\lambda)\, e^{-s\lambda}\, d\lambda = \int_0^{\infty}\sum_k^{\infty} g(kT)\,\delta(\lambda-\kappa T)\, e^{-s\lambda}\, d\lambda$$

$$= \sum_k^{\infty}\int_0^{\infty} g(kT)\, e^{-s\lambda}\,\delta(\lambda-\kappa T)\, d\lambda = \sum_k^{\infty} g(kT)\, e^{-kTs},$$

then in a similar way one can obtain

$$U^+(s) = \int_0^\infty u^+(t)\, e^{-st}\, dt = \sum_k^\infty u(kT)\, e^{-kTs}$$

$$Y^+(s) = \int_0^\infty y^+(t)\, e^{-st}\, dt = \sum_k^\infty y(kT)\, e^{-kTs}$$

Based on the definition of the Z-transformation and comparison of (1-6) and (1-8), we have

$$G^+(s) = G(z)\Big|_{z=e^{sT}} = G(e^{sT})$$

$$U^+(s) = U(z)\Big|_{z=e^{sT}} = U(e^{sT}) \qquad (1\text{-}8)$$

$$Y^+(s) = Y(z)\Big|_{z=e^{sT}} = Y(e^{sT})$$

Because

$$e^{(s+jn\omega_s)T} = e^{sT}\, e^{jn\omega_s T} = e^{sT}$$

$\omega_s = 2\pi/T$ T -- sampling period

we have

$$G^+(s + jn\omega_s) = G^+(s) \qquad (1\text{-}9)$$

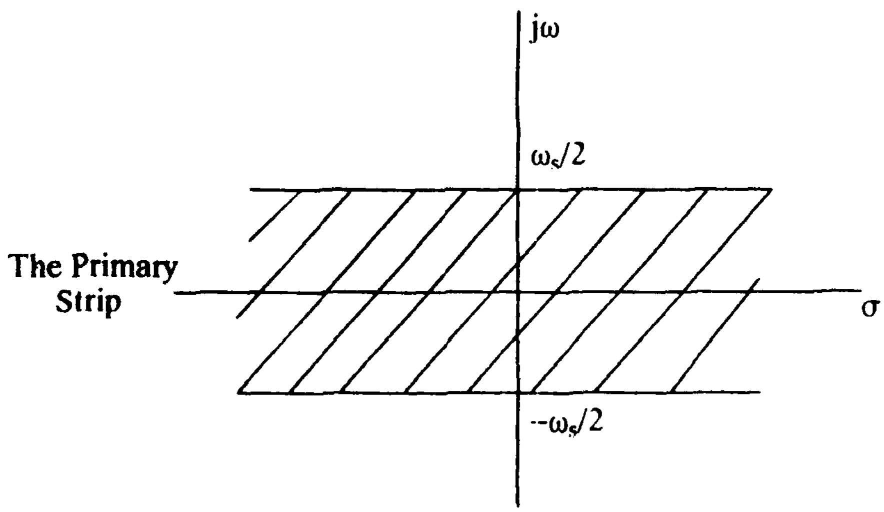


Fig. 1-4 The Frequency Domain for the Discrete-time System

The following transfer function properties are limited to a particular frequency range, indicated by the primary strip in Fig. 1–4

$$G^+(j\omega) = G(e^{2\pi j\omega/\omega_s}) \qquad (\omega_s / 2 < \omega < \omega_s / 2) \qquad (1\text{–}10)$$

$$G^+(j\omega) = G(e^{j\theta}) \qquad (-\pi < \theta < \pi) \qquad (1\text{–}11)$$

1.4 The relationship between the input–output for the equivalent continuous–time system and the discrete–time system

Because the output of the equivalent continous–time systems is

$$y^+(t) = L^{-1}\{ Y^+(s) \} = L^{-1}\{G^+(s)\ U^+(s)\} = \int_0^\infty g^+(t-\tau)\ u^+(\tau)\ d\tau$$

$$= \int_0^\infty \sum_k^\infty g(kT)\ \delta(t-\tau-kT)\ \sum_m^\infty u(mT)\ \delta(\tau-mT)\ d\tau$$

$$= \sum_k^\infty \sum_m^\infty g(kT)\ u(mT) \int_0^\infty \delta(t-\tau-kT)\ \delta(\tau-mT)\ d\tau$$

$$= \sum_k^\infty \sum_m^\infty g(kT)\ u(mT)\ \delta(t-mT-kT)$$

$$= \sum_k^\infty \sum_m^\infty g(kT)\ u(mT)\ \delta(t-nT) \qquad (m+k=n)$$

$$= \sum_m^\infty \sum_n^\infty g((n-m)T)\ u(mT)\ \delta(t-n\tau) \qquad (g((n-m)T)=0,\ n<m)$$

$$= \sum_n^\infty y(nT)\ \delta(t-nT)$$

where

$$y(nT) = \sum_m^\infty g((n-m)T)\ u(mT)$$

It shows that the relationship between the output–input of the equivalent continuous–time system is similar to that of the discrete–time system.

1.5 Summary

(1) If the transfer function of the discrete-time system is given by $G(z)$, then the transfer function of its equivalent continuous-time system, $G^{+}(s)$, can be expressed as follows:

$$G^{+}(s) = G(z)\Big|_{z=e^{sT}}$$

$$G^{+}(j\omega) = G(e^{j\theta}) \qquad (-\pi < \theta < \pi)$$

$$= G(e^{2\pi j\omega/\omega_s}) \qquad (-\omega_s/2 < \omega < \omega_s/2)$$

(2) The frequency properties for the discrete-time system can be described in terms of frequncy properties for the equivalent continuous-time system.

2 Robustness Criterion in the Frequency Domain for Discrete–Time Systems with Additive Alterations or with Multiplicative Alterations

2.1 The basic theorem of stability for MIMO linear feedback system

We will use the standard nota ion of input–output stability theory[7]

$\mathfrak{B}$ = some Banach space of function $x : T \to X$ with $\| \cdot \|$
T = subset of the real numbers
X = finite dimensional vector space
$\mathfrak{B}_e = \{ x : P_\tau x \in \mathfrak{B} \text{ for all } \tau \in T \}$

$$(P_\tau x)(t) = \begin{cases} x(t) & t \leqslant \tau \\ 0 & t > \tau \end{cases}$$

$\mathcal{L}_2^m$ = space of m–vector function on T with integrable Euclidean norm

$\mathfrak{I} : \mathfrak{B}_e \to \mathfrak{B}_e$ = identity operator
$\mathcal{G} : \mathfrak{B}_e \to \mathfrak{B}_e$ is causal if $P_\tau \mathcal{G} P_\tau = P_\tau \mathcal{G}$ for all $\tau \in T$

A^* = conjugate transpose of a complex matrix A

$$\| \mathcal{G} \| \underset{\Delta}{=} \sup_{\substack{x_1, x_2 \in \mathfrak{B}_e \\ P_\tau x_1 \neq P_\tau x_2}} \frac{\| P_\tau \mathcal{G} x_1 - P_\tau \mathcal{G} x_2 \|}{\| P_\tau x_1 - P_\tau x_2 \|} \qquad \tau \in T$$

We consider the feedback system depicted in Fig.2–1. Here the causal operator $\mathcal{G}$: $\mathcal{L}_{2e}^m \to \mathcal{L}_{2e}^m$ represents the plant plus the any compensator that is used.

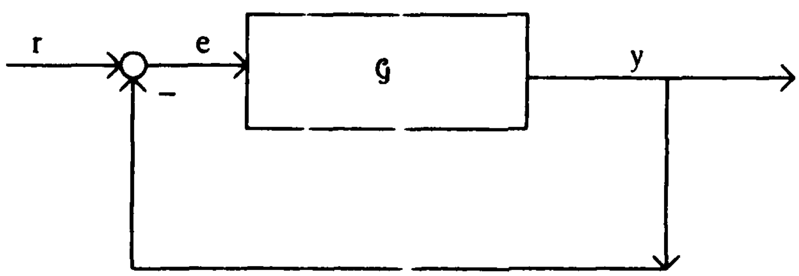


Fig. 2–1 Basic MIMO Linear Feedback System

The basic feedback equation is

$$(\mathcal{I} + \mathcal{G})e = r \tag{2-1}$$

and the basic stability question is whether $(\mathcal{I} + \mathcal{G})^{-1} : \mathcal{L}_{2e}^m \longrightarrow \mathcal{L}_{2e}^m$ exists, is causal, and is a bounded operator in the sense that $\|\,(\mathcal{I} + \mathcal{G})^{-1}\| < \infty$. We will assume that the nominal system is stable. We are interested in whether the closed loop sysem retains these properties when subject to additive ($\mathcal{G} \rightarrow \mathcal{G} + \Delta\mathcal{G}$) or multiplicative ($\mathcal{G} \rightarrow \mathcal{G} + \mathcal{G}\,\Delta\mathcal{G}$) perturbations representing uncertainty in the dynamic behavior of the system. The following theorem provide the basis for our analysis .[7]

Theorem 1 : $\mathcal{A} : \mathfrak{B}_e \longrightarrow \mathfrak{B}_e$ be a linear causal operator, and suppose $\mathcal{A}^{-1}$ exists, is causal and $\|\mathcal{A}^{-1}\|_{\Delta} < \infty$. Then if $\Delta\mathcal{A} : \mathfrak{B}_e \longrightarrow \mathfrak{B}_e$ is a causal operator satisfying $\|\Delta\mathcal{A}\|_{\Delta} < \infty$ and if

$$\|\mathcal{A}^{-1}\Delta\mathcal{A}\|_{\Delta} < 1 \tag{2-2}$$

it follows that $(\mathcal{A}+\Delta\mathcal{A})^{-1} : \mathfrak{B}_e \longrightarrow \mathfrak{B}_e$ exists, is causal and has

$$\|(\mathcal{A}+\Delta\mathcal{A})^{-1}\|_{\Delta} < \infty \tag{2-3}$$

Theorem 2 : (Desoer and Vidyasager 1975 [4]) Let the operator $\mathcal{G} : \mathcal{L}_{2e}^m \longrightarrow \mathcal{L}_{2e}^m$ for $T = [0, \infty]$ be defined by

$$(\mathcal{G}x)(t) = \int_0^{\infty} \mathcal{G}(t - \tau)x(\tau)\, d\tau \tag{2-4}$$

where the elements of the impulse response matrix $\mathcal{G}(t)$ are assumed absolutely integrable on T . Then

$$\|\mathcal{G}\|_{\Delta} = \|\mathcal{G}\|_{\mathcal{L}_{2e}^m} = \sigma_{max} \tag{2-5}$$

where

$$\sigma_{max} = \max_{\omega>0,\, 1\leq i\leq m} \sigma_i(G(j\omega))$$

and where $\sigma_i(G(j\omega))$ denotes the ith singular value of the transfer function matrix conresponding to $\mathcal{G}$.

2.2 The stability conditions for the MIMO feedback system with additive alteration or with multiplicative alteration

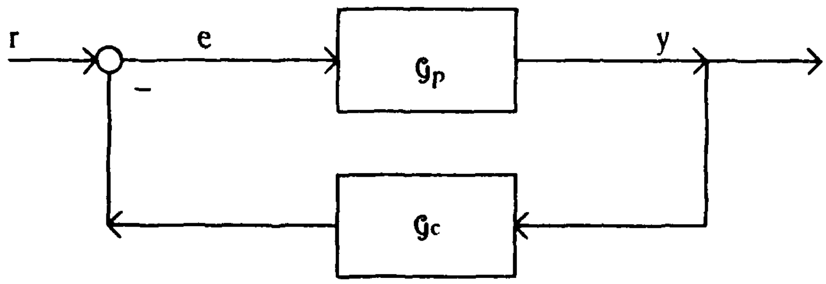


Fig. 2–2 Nomial Feedback System

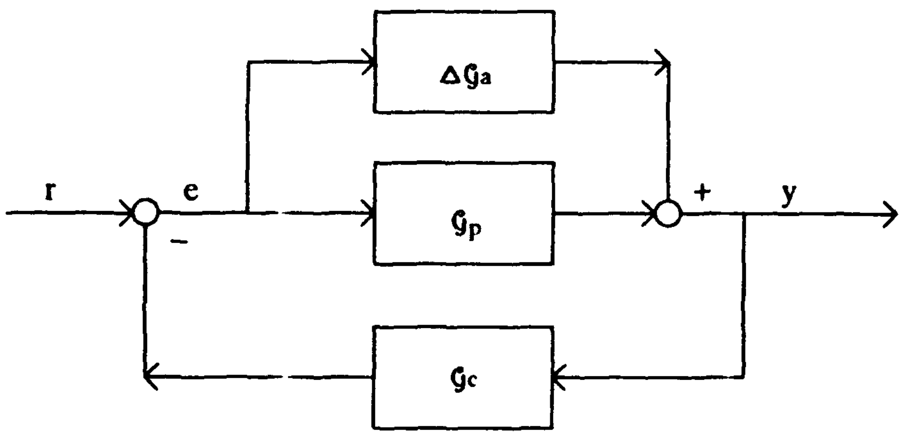


Fig. 2–3 Feedback System with Additive Alteration

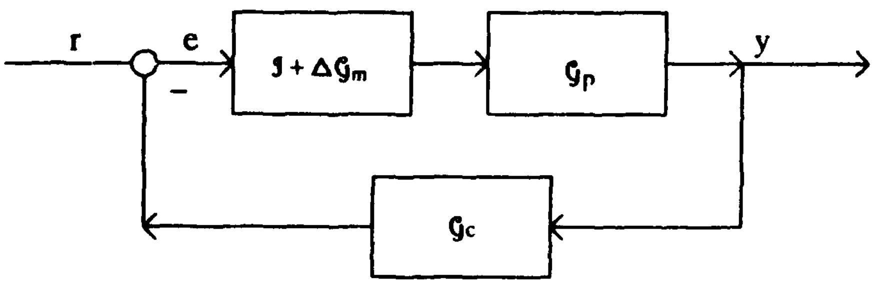


Fig. 2–4 Feedback System with Multiplicative Alteration

Where

$\mathcal{G}_p : \mathcal{L}_{2e}^m \longrightarrow \mathcal{L}_{2e}^m$ the causal operator of the plant

$\mathcal{G}_c : \mathcal{L}_{2e}^m \longrightarrow \mathcal{L}_{2e}^m$ the causal operator of the compensator

$\Delta\mathcal{G}_a : \mathcal{L}_{2e}^m \longrightarrow \mathcal{L}_{2e}^m$ the causal operator of the additive alteration

$\Delta\mathcal{G}_m : \mathcal{L}_{2e}^m \longrightarrow \mathcal{L}_{2e}^m$ the causal operator of the multiplicative alteration

The coresponding transfer functions are G_p, G_c, ΔG_a, ΔG_m, respectively.

2.2.1 Feedback system with additive alteration

For the feedback system with additive alteration, Fig.2–3, we have

$$y=(\mathcal{G}_p e + \Delta\mathcal{G}_a e) = (\mathcal{G}_p + \Delta\mathcal{G}_a) e$$

$$e=r-\mathcal{G}_c y$$

i.e.,
$$r=(\mathcal{I} + \mathcal{G}_c \mathcal{G}_p + \mathcal{G}_c \Delta\mathcal{G}_a)e$$

Applying the Theorem 1, if

$$||(\mathcal{I} + \mathcal{G}_c\mathcal{G}_p)^{-1}\mathcal{G}_c\Delta\mathcal{G}_a||_{\Delta} < 1 \qquad (2-6)$$

then

$$(\mathcal{I} + \mathcal{G}_c\mathcal{G}_p + \mathcal{G}_c\Delta\mathcal{G}_a)^{-1} \text{ exist and is bounded}$$

Applying Theorem 2 to (2–6), and considering the relationship between the discrete–time system and the equivalent continuous–time system, we have the following sufficient condition for system stability:

$$||(I+G_c(e^{2\pi j\omega/\omega_s})G_p(e^{2\pi j\omega/\omega_s}))^{-1}G_c(e^{2\pi j\omega/\omega_s})\Delta G_a(e^{2\pi j\omega/\omega_s})||_2 < 1$$

$$\forall\, \omega \in [\, 0,\ \omega_s/2\ ) \qquad (2-7)$$

Because

$$\bar{\sigma}(A) = \| A \|_2 , \qquad \underline{\sigma}(A) = (\| A^{-1} \|_2)^{-1}$$

we can obtain (2–8) from (2–7)

$$\| (I+GcGp)^{-1}Gc\Delta Ga \|_2 < \| (I+GcGp)^{-1}Gc \|_2 \ \| \Delta Ga \|_2 < 1 \qquad (2-8)$$

If G_c^{-1} exists, then

$$\| \Delta G_a \|_2 < \|(I+G_cG_p)^{-1}G_c \|_2^{-1} = \|(G_c^{-1} + G_p)^{-1}\|_2^{-1}$$

i.e.,

$$\bar{\sigma}[\Delta G_a(e^{2\pi j\omega/\omega_s})] < \underline{\sigma}[G_c^{-}(e^{2\pi j\omega/\omega_s}) + G_p(e^{2\pi j\omega/\omega_s})] \qquad \forall\omega \in [0, \omega_s/2) \quad (2-9)$$

or we have from (2-8)

$$\|(I+G_cG_p)^{-1}\|_2 \|G_c\|_2 \|\Delta G_a\|_2 < 1$$

i.e.,

$$\bar{\sigma}[\Delta G_a(e^{2\pi j\omega/\omega_s})] < \frac{\underline{\sigma}[I \cdot G_c(e^{2\pi j\omega/\omega_s})G_p(e^{2\pi j\omega/\omega_s})]}{\underline{\sigma}[G_c(e^{2\pi j\omega/\omega_s})]} \qquad \forall\, \omega \in [\, 0, \omega_s) \quad (2-10)$$

2.2.2 Feedback system with multiplicative alteration

From Fig.(2-4), we have

$$y = \mathcal{G}_p(\mathcal{I} + \Delta\mathcal{G}_m)e$$

$$e = r-\mathcal{G}_c y$$

i.e.,

$$r = (\mathcal{I} + \mathcal{G}_c \mathcal{G}_p + \mathcal{G}_c \mathcal{G}_p \Delta\mathcal{G}_r)e$$

Applying Theorem 1, if

$$\| (\mathcal{I} + \mathcal{G}_c\mathcal{G}_p)^{-1} \mathcal{G}_c \mathcal{G}_p \Delta\mathcal{G}_m \|_{\Delta} < 1 \qquad (2-11)$$

then

$(\mathcal{I} + \mathcal{G}_c\mathcal{G}_p + \mathcal{G}_c\mathcal{G}_p\Delta\mathcal{G}_m)^{-1}$ exists and is bounded.

Applying Theorem 2 to (2-11) and considering the relationship between the discrete-time system and the equivalent continuous-time system, we have the following sufficient condition for system stability:

$$\| (I+G_c^+(j\omega)G_p^+(j\omega))^{-1}G_c^+(j\omega)G_p^+(j\omega)\Delta G_m^+(j\omega)\|_2 < 1 \qquad \omega_s = 2\pi/T$$

i.e.,

$$\| (I+G_c(e^{2\pi j\omega/\omega_s})G(e^{2\pi j\omega/\omega_s}))^{-1}G_c(e^{2\pi j\omega/\omega_s})G_p(e^{2\pi j\omega/\omega_s})\Delta C_m(e^{2\pi j\omega/\omega_s})\|_2 < 1$$

$$\forall\, \omega \in [0.\omega_s/2) \qquad (2\text{–}12)$$

We may obtain the following relationship from (2- 12)

$$\| (I+G_cG_p)^{-1}G_pG_c\Delta G_m)\|_2 < \|(I+G_cG_p)^{-1}G_cG_p\|_2 \ \|\Delta G_m\|_2 < 1$$

i.e.,

$$\bar{\sigma}[\Delta G_m] < 1/\bar{\sigma}[(I+G_cG_p)^{-1}G_cG_p] \qquad (2\text{–}13)$$

If $(G_cG_p)^{-1}$ exists, then

$$\bar{\sigma}[\Delta G_m] < \underline{\sigma}[\ (G_cG_p)^{-1} + I\ ]$$

i.e.,

$$\bar{\sigma}[\Delta G_m(e^{2\pi j\omega/\omega_s})] < \underline{\sigma}[(G_c(e^{2\pi j\omega/\omega_s})G_p(e^{2\pi j\omega/\omega_s}))^{-1} + I\ ] \qquad (2\text{–}14)$$

$$\forall\, \omega \in [0,\ \omega_s/2)$$

Because

$$(I+A)+(I+A^{-1}) = (I+A)(I+A^{-1})$$

then

$$(I+A)^{-1}+(I+A^{-1})^{-1} = I$$

$$\|(I+A^{-1})^{-1}\|_2 - \|(I+A)^{-1}\|_2 \leqslant 1$$

i.e.,

$$1/\underline{\sigma}[I+A^{-1}] \leqslant 1 + 1/\underline{\sigma}[I+A]$$

i.e.,

$$\underline{\sigma}[I+A^{-1}] \leqslant \underline{\sigma}[I+A]/(1+\underline{\sigma}[I+A]) \qquad (2\text{–}15)$$

Considering (2-14) and (2-15), we obtain (2-16) if $(G_cG_p)^{-1}$ does not exist

$$\bar{\sigma}[\Delta G_m(e^{2\pi j\omega/\omega_s})] < \frac{\underline{\sigma}[\ I+G_c(e^{2\pi j\omega/\omega_s})G_p(e^{2\pi j\omega/\omega_s})]}{I + \underline{\sigma}[I+G_c(e^{2\pi j\omega/\omega_s})G_p(e^{2\pi j\omega/\omega_s})]}$$

$$< \underline{\sigma}[(G_cG_p)^{-1}+ I] \qquad \forall\, \omega \in [0,\ \omega_s/2) \qquad (2\text{-}16)$$

3 The Linear-Quadratic-Gaussian(LQG)/Loop-Transfer-Recovery(LTR) Methodology for the Discrete-time System

3.1 Introduction

One of the most important breakthroughs in multi-input, multi-output feedback system theory for the last decade is the development of the loop transfer recovery methodology for the linear quadratic Gaussian problem, which is called LQG/LTR. The people who developed this techniques at first are Kwakernaak, Doyle and Stein [1,2]

These are design techniques which allow the excellent robustness and sensitivity properties of optimal state feedback schemes to be almost recovered by output feedback schemes. Although this was the original motivation for the development of these techniques, a wider and more important aspect of them is that they simplify the use of LQG methodology, allowing practical feedback designs to be attained with a reasonable amount of effort.

Whereas output feedback design via LQG methods usually requires the specification of two pairs of matrices, namely a pair of cost-weighting matrices and a pair of noise covariance matrices, the asymptotic recovery approach requires only one of these pairs to be designed, with the other pair being assigned values according to an automatic procedure. This results in a tremendous reduction in the complexity of the design process. Consequently, it is of great importance to obtain an analogous procedure for discrete-time systems. Unfortunately, the previous research work in this field is almost all for the continuous-time system (except Maciejowski who is with Cambridge University, England) The difficulty of the robustness recovery problem (or sensitivity recovery) for the discrete-time system is that the filter gain (or control gain) for the discrete-time system is finite, but the filter gain (or control gain) for the continuous-time system is infinite, so robustness (or sensitivity) recovery can not be obtained by simple reduction similar to the case for the continuous time system.

Maciejowski's work[10]is only about the development of the sensitivity recovery of LQG control with the filter observer for the discrete-time system. But with discrete-time systems, the duality with the optimal control is only for the predicting observer,not for the filter observer. Therefore, the robustness recovery for the discrete-time system can not be obtained by simply using the duality principle.

Our research work is to develop techniques for ensuring robustness recovery for discrete-time systems.The specific developments are as follows.

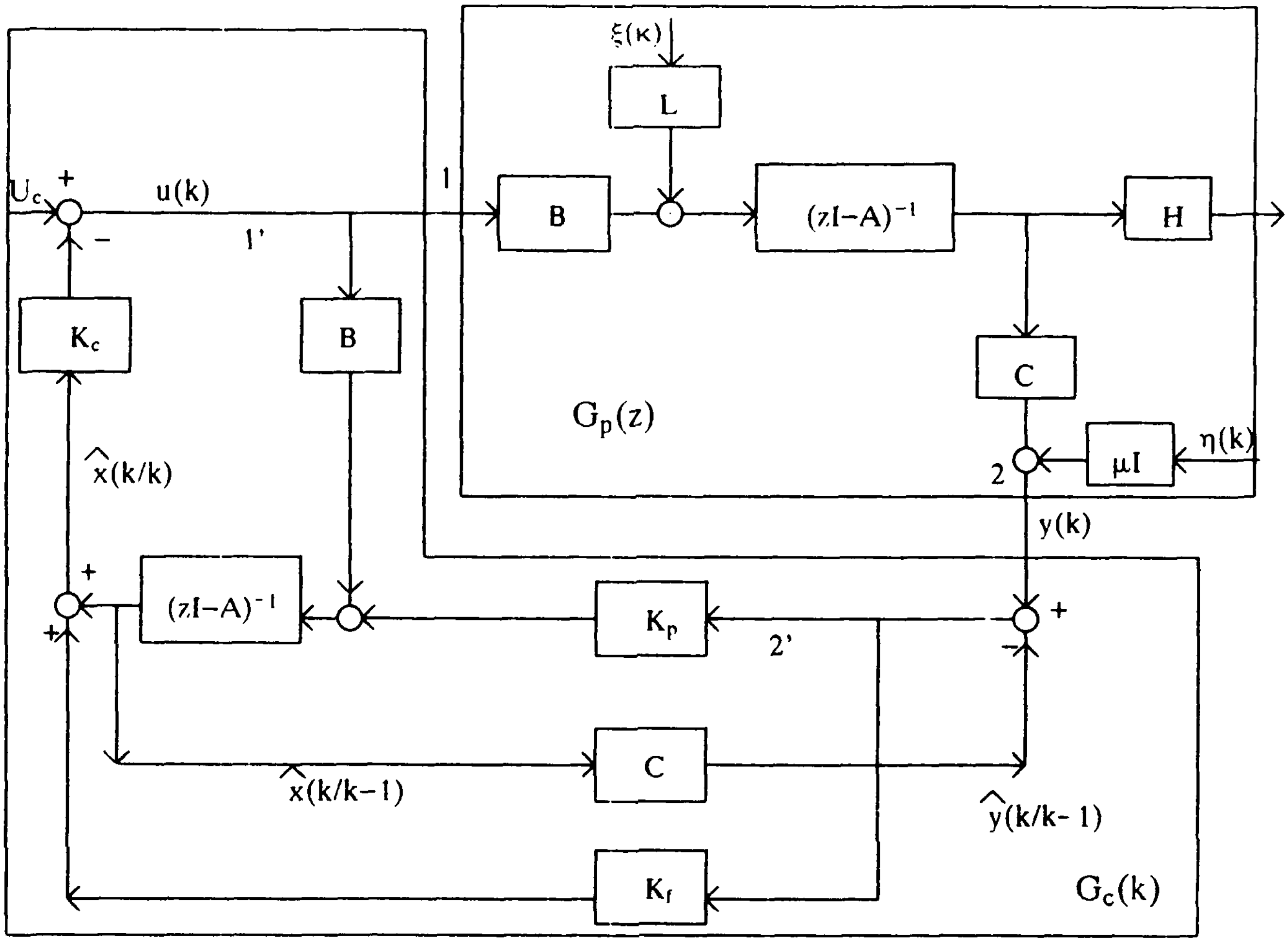


Fig.3−1 Block Diagram of the LQG Digital Control with Filtering Observer

where the transfer functions of the plant and componsentor are $G_p(z)$ and $G_c(z)$, they are as follows:

$$G_p(z)=C(zI-A)^{-1}B$$

$$G_c(z)=zK_c[zI-(I-K_fC)(A-BK_c)]^{-1}K_f$$

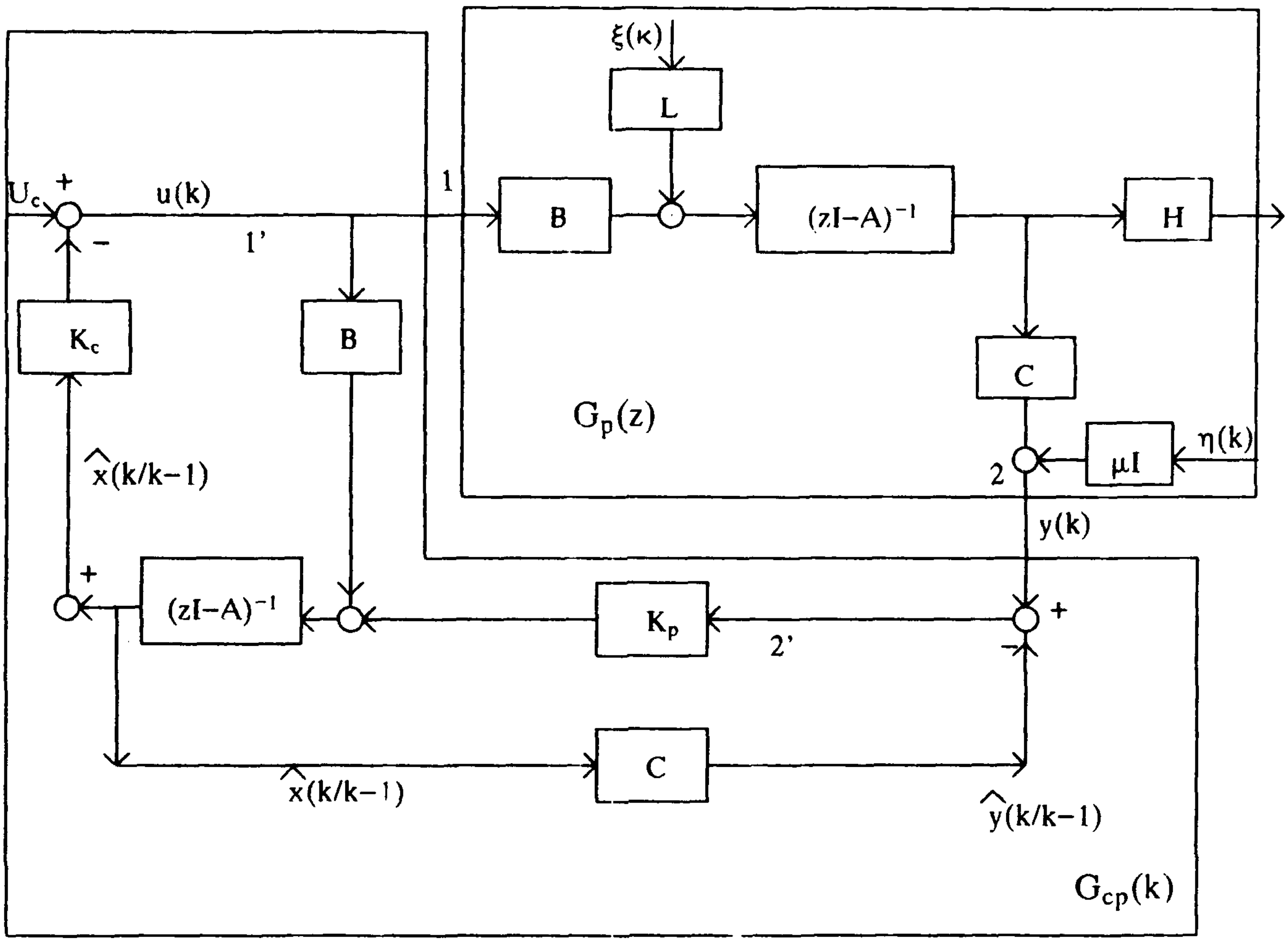


Fig.3–2 Block Diagram of the LQG Digital Control with Predicting Observer

where the compensator transfer function for the predicting observer, $G_{cp}(z)$, is as follows as:

$$G_{cp}(z) = K_c(zI-A_k)^{-1}K_p$$

$$A_k = A - BK_c - K_pC$$

3.2 LQG/LTR Methodology for Discrete–time Systems

3.2.1 The statement of the LQG problem

It is given that the system state equation is

$$x(k+1)=Ax(k)+Bu(k)+L\xi(k) \tag{3-1}$$

The measurement equation of the system is

$$y(k)=Cx(k)+\mu I\eta(\kappa) \tag{3-2}$$

The control output equation is

$$y_c(k)=Hx(k) \tag{3-3}$$

It is assumed that the statistical properties are given by

$$E\{\xi(\kappa)\xi(\kappa)^T\} = I \qquad E\{\eta(\kappa)\eta(\kappa)^T\} = I \tag{3-4}$$

then

$$E\{(L\xi(k))(L\xi(k))^T\} = LL^T = Q \tag{3-5}$$

$$E\{(\mu I\eta(\kappa))(\mu I\eta(\kappa))^T\} = \mu^2 I = R \tag{3-6}$$

where

- L   noise input matrix
- μ parameter of measurement noise
- H   control output matrix
- ρ parameter of control weighting matrix

The objective of the LQG problem is to find a controller depending only on y(k), u(k) (k=1,2,3,...), to minimize the performance index, J, where

$$J = E\{ \sum_{k=0}^{\infty} (x^T(k)\hat{Q}x(k) + u^T(k)\hat{R}u(k) \} \tag{3-7}$$

where

$$H^T H = \hat{Q} \qquad \rho^2 I = \hat{R} \tag{3-8}$$

It is well known that if the system (A B H) is controllable and observable and the system

(A L C) is controllable and observable(the conditions may be reduced to stabilizable and detectable for the time-invariant system), then the closed-loop system of the LQG optimal controller is asymptotically stable. It should be pointed out that the free parameters of this problem are L (noise input matrix), μ (intensity of the observational noise), H (control output matrix), and ρ(control weighting factor in the performance index). In typical LQG applications, these parameters are assigned a priori physical significance (e.g., process noise, sensor noise, controlled variables, and control weights). Then the solutions for the LQG problem are :

$$u(k) = -K_c \hat{x}(k/k) \qquad \text{(For the filter observer)} \tag{3-9}$$

or

$$u(k) = -K_c \hat{x}(k/k-1) \qquad \text{(For the predictor observer)} \tag{3-10}$$

where

$$\hat{x}(k/k)=\hat{x}(k/k-1)+K_f(y(k)-\hat{y}(k/k-1)) \tag{3-11}$$

$$\hat{x}(k+1/k)=A\hat{x}(k/k-1)+Bu(k)+K_p(y(k)-\hat{y}(k/k-1)) \tag{3-12}$$

$$\hat{y}(k/k-1)=C\hat{x}(k/k-1) \tag{3-13}$$

$$K_p=AK_f \tag{3-14}$$

$$K_f=P_eC^T(CP_eC^T+R)^{-1} \tag{3-15}$$

$$P_e=AP_eA^T-AP_eC^T(R+CP_eC^T)^{-1}CP_eA^T+Q \tag{3-16}$$

and

$$K_c=(\hat{R}+B^TPB)^{-1}BPA \tag{3-17}$$

$$P=A^TPA-A^TPB(\hat{R}+B^TPB)^{-1}B^TPA+\hat{Q} \tag{3-18}$$

The block diagrams of the optimal LQG digital control with filter observer and with predicting observer are shown in Fig. 3-1and Fig. 3-2, respectively.

3.2.2 The development of the transfer function matrices

From (3-11) and (3-13), we have

$$\hat{x}(k/k)=(I-K_fC)\hat{x}(k/k-1)+K_fy(k) \tag{3-19}$$

Considering (3-9), (3-12), and (3-13), we have

$$\hat{x}(k+1/k)=(A-BK_c)(I-K_fC)\hat{x}(k/k-1)+(A-BK_c)K_fy(k) \tag{3-20}$$

Let $x^-(z)=Z\{\hat{x}(k/k-1)\}$, $x(z)=Z\{\hat{x}(k/k)\}$, $y(z)=Z\{y(k)\}$ (3-21)

The Laplace transformation of (3-20) can be written as follows

$$x^-(z)=[zI-(A-BK_c)(I-K_fC)]^{-1}(A-BK_c)K_fy(z) \tag{3-22}$$

The Laplace transformation of (3-19) is

$$\begin{aligned} x(z)&=(I-K_fC)x^-(z)+K_fy(z) \\ &=(I-K_f)[zI-(A-BK_c)(I-K_fC)]^{-1}(A-BK_c)K_fy(z)+K_fy(z) \\ &=\{(I-K_fC)[zI-(A-BK_c)(I-K_fC)]^{-1}(A-BK_c)+I\}K_fy(z) \end{aligned} \tag{3-23}$$

Because of

$$x(zI-yx)^{-1}y+I=z(zI-xy)^{-1} \tag{3-24}$$

then (3-23) can be written as follows:

$$x(z)=z[zI-(I-K_fC)(A-BK_c)]^{-1}K_fy(z) \tag{3-25}$$

Let $G_p(z)$ be the transfer function of the plant;
$G_c(z)$ be the transfer unction of the compensator with filtering obserber;
$G_{cp}(z)$ be the transfer function of the compensator with predicting observer.

Then the compensator transfer function for the filtering observer, $G_c(z)$, can be obtained from(3-9) and (3-25) as :

$$G_c(z)=zK_c[zI-(I-K_fC)(A-BK_c)]^{-1}K_f \tag{3-26}$$

The $G_p(z)$ can be obtained from (3-1)and (3-2):

$$G_p(z)=C(zI-A)^{-1}B \tag{3-27}$$

The compensator transfer function for the predicting observer, $G_{cp}(z)$, can be obtained from (3-10),(3-12),and(3-13) and is expressed :

$$G_{cp}(z)=K_c(zI-A_k)^{-1}K_f \tag{3-28}$$

where

$$A_k=A-BK_c-K_pC \tag{3-29}$$

3.2.3 Robustness recovery and sensitivity recovery

It is well known that the multivariable linear–quadratic(LQ) optimal regulators have impressive robustness properties, including guaranteed classical gain margins of −6 db to +∞ db and phase margins of +60° in all channels. The result is only valid, however, for the full–state case. If observer or Kalman filters are used in the implementation, no guaranteed robustness properties hold. The robustness recovery means that if the measurement noise parameter, μ, approaches zero, then the loop transfer function of the LQG control at the input loop–breaking point 1, will approach the loop transfer function of the LQR control. The sensitivity recovery means that the loop transfer function of the LQG control at the output loop–breaking point 2, will approach the loop transfer function of the Kalman filter when the weighting parameter, ρ, approaches zero[. −3]. We will prove that these results are also true for discrete–time systems.

The following facts can be found from Fig.3−1:

(a) The loop transfer function obtained by breaking the LQG loop at point, 1', is the LQR loop transfer function $T_1(z)$,i.e.,$K_c(zI-A)^{-1}B$:

(b) The loop transfer function obtained by breaking the LQG loop at point, 1, is G_cG_p;

(c) The loop transfer function obtained by breaking the LQG loop at point, 2', is the Kalman filter transfer function,$T_3(z)$, i.e., $C(zI-A)^{-1}K_p$;

(d) The loop transfer function obtained by breaking the LQG loop at point, 2, is G_pG_c.

If the discrete–time system also has the properties of the loop transfer recovery similar to the case for the continuous–time system, then the following relationships should be satisfied.

$$\lim_{\mu \to 0} G_cG_p = K_c(zI-A)^{-1}B \qquad \text{(Robustness recovery)} \qquad (3-30)$$

and

$$\lim_{\rho \to 0} G_pG_c = C(zI-A)^{-1}K_p \qquad \text{(Sensitivity recovery)} \qquad (3-31)$$

Before beginning the formal proof, a lemma will be used:

Lemma (Shaked[11]) : If det(CB) $\neq$ 0, and system (3−1)−(3−3) is of minimum phase, then the Kalman filter gain K_p determined by (3−14)−(3−16) will be $AB(CB)^{-1}$ when the variance parameter of the observational noise, μ, approaches to zero, i.e.,

$$\lim_{\mu \to 0} K_p = AB(CB)^{-1}$$

This is the simplest case of Shaked's much more general result. Because $K_p=AK_f$ and null(A)=0, therefore Limt $K_f=B(CB)^{-1}$.

Theorem: If the open loop transfer function of the system (3–1)–(3–3), has no (finite) zero in { z : |z| > 1 } and det(CB) ≠ 0, then

$$\lim_{\mu \to 0} G_c G_p = K_c(zI-A)^{-1}B \tag{3-32}$$

Proof:

Let $\Delta(z) = G_cG_p - K_c(zI-A)^{-1}B$

$$\lim_{\mu\to 0} \Delta(z) = \lim_{\mu\to 0} G_cG_p - K_c(zI-A)^{-1}B$$

$$= \lim_{\mu\to 0} zK_c[zI-(I-K_f)(A-BK_c)]^{-1}K_fC(zI-A)^{-1}B - K_c(zI-A)^{-1}B$$

$$= K_c\{\lim_{\mu\to 0} z[zI-(I-K_fC)(A-BK_c)]^{-1}K_fC-I\}(Iz-A)^{-1}B$$

$$= K_c\{z[zI-(I-S)(A-BK_c)]^{-1}S-I\}(zI-A)^{-1}B$$

$$= K_c\{z[zI-(I-S)A]^{-1}S-I\}(zI-A)^{-1}B$$

$$= K_c[zI-(I-S)A]^{-1}\{zS-[zI-(I-S)A]\}(zI-A)^{-1}B$$

$$= K_c[zI-(I-S)A]^{-1}\{z(S-I)+(I-S)A\}(zI-A)^{-1}B$$

$$= -K_c[zI-(I-S)A]^{-1}(I-S)(zI-A)(zI-A)^{-1}B$$

$$= -K_c[zI-(I-S)A]^{-1}(I-S)B$$

$$= 0 \qquad (\text{ Since } (I-S)B=0 \text{ }) \tag{3-33}$$

where

$$S = I - B(CB)^{-1}C$$

The satisfaction at Eq.(3–32) means that there is the robustness recovery properties for the digital LQG control with filtering observer, but there is no the robustness recovery properties for the digital LQG control with the predicting observer, because the G_c and G_{cp} is different, (3–32) can not be proven by using G_{cp} instead of the G_c.

In similar way the (3–31) can be also proven, it indicate that the sensitivity recovery is also true for the digital LQG control with the filtering observer.

Note:

1. The condition, $\det(CB) \neq 0$, ensures that $G_p(z)$ has the maximum possible number $(n-m)$ of finite zeros, and the minimum possible number (m) of infinite zeros. The perfect recovery obtained is only possible because the nonzero poles of G_c cancel the $(n-m)$ finite zeros of $G_p(z)$, and the m origin poles of $G_p(z)$ cancel the m origin zeros introduced by the factor z in (3-26).

2. The mechanism by which the recovery is achieved is essentially the same as in the continuous-time system case : the compensator cancels the plant zeros and possibly some of the stable poles, and inserts the controller (observer) zeros. Clearly, this will fail if the plant has zeros outside the unit circle, since the compensator, G_c, guarantees internal stability.

4 Design of Reduced Order Optimal Digital LQG Controller for the Orbiting Flexible Shallow Spherical Shell System

4.1. Introduction

Future proposed space miss ons would involve large inherently flexible systems for use in communications, radiometry, and in electronic orbital based mail systems. The use of very long shallow dish-type structures to be employed as receivers/reflectors for these missions has been suggested. In order to satisfy mission requirements, the proposed LQG digital optimal control of the shape and orientation for an orbiting shallow spherical shell are also studied [1].

Since the mathematical system model is inherently of high order, a practical controller has to be based on a reduced order design model. It is the purpose of this chapter to study the design of the reduced order optimal digital controller for the orbiting shallow spherical shell system.

One of the most important breakthroughs in multi-input, multi-output feedback system theory for the last decade is the development of the loop transfer recovery methodology for the linear quadratic Gaussian technique, which is called LQG/LTR [6-8]. These modern techniques are very useful in treating unmodeled dynamics and stochastic uncertainties such as disturbances and sensor noises. As we know, these methods have been developed only for the continuous-time system. However, in practice, observational data used to verify the orientation and shape of large space flexible systems will, in general, be collected on a sampled basis (discrete-time data system). In order to meet the requirements of design for the discrete-time data system, the loop transfer recovery problem for the discrete-time system has been developed in our current research work; It has been proven that the robustness recovery property for the LQG problem is also true for the discrete-time system if the open loop transfer function of the system has no (finite) zeros outside the unit circle and det(CB)=0. where the C is the observation matrix, and B is the control influence matrix.

The main purpose of this chapter is not to show how to develop this method theoretically in detail from the continuous-time system to the discrete-time system(this will appear in another paper), but to apply these results to the design of robust reduced-order LQG controllers for large space structural systems with sampled input data.

4.2 Mathematical Models

The mathematical model of an isotropic shallow flexible spherical shell in orbit was developed in Refs [14-16]. The resulting linearized equations of motion for the rigid rotational and generic elastic modes were developed as :

$$\ddot{\psi} - \mu_1\psi - (1+\mu_1)\dot{\phi} = C_x/J_x^{(0)}\omega_c^2$$

$$\ddot{\phi} + 4\mu_3\phi + (1-\mu_3)\dot{\psi} = C_z/J_z^{(0)}\omega_c^2 \qquad (4-1)$$

$$\ddot{\theta} - 3\mu_2\theta - (2l/J_y^{(0)}) \sum_{n=1}^{10} I_1^{(n)}\dot{\epsilon}_n = C_y/J_y^{(0)}\omega_c^2$$

$$\ddot{\epsilon}_i + (\Omega_i^2 - 3)\epsilon_i + (2I_1^{(i)}/M_i l)\dot{\theta} = 3I_1^{(i)}/M_i l + E_i/M_i l\omega_c^2 \qquad (i = 1,2,3,\ldots 10)$$

where

$$\tau = \omega_c t, \qquad \epsilon_i = q_i(t)/l \qquad (i=1,2,3,\ldots)$$

The derivative in Eq.(4−1) is with respect to τ.

ψ,ϕ,θ = yaw, roll, and pitch angles, respectively, between the undeformed axes of the shell and the axis of the orbiting local vertical /local horizontal system.

$q_i(t)$ = modal amplitude of the ith generic mode.

ω_c = orbital angular rate, constant for assumed circular orbit.

l= charateristic length (the base radius).

M_i = the ith modal mass.

$J_x^{(0)}, J_y^{(0)}, J_z^{(0)}$ = principal moments of inertia of the undeformed shell.

C_x, C_y, C_z = the components of external torques.

x_c = coordinate of differential area on the surface above the base plane.

$\mu_1, \mu_2, \mu_3 = (J_z^{(0)}-J_y^{(0)})/J_x^{(0)}, (J_x^{(0)}-J_z^{(0)})/J_y^{(0)}, (J_y^{(0)}-J_x^{(0)})/J_z^{(0)}$, respectively.

$I_1^{(i)} = \int_\nu x_c \Phi_x^{(i)} d\nu$

$\Phi_x^{(n)}$ = transverse component of the nth modal shape function.

ω_n = natural frequency of nth mode.

$\Omega_n = \omega_n/\omega_c$

The mode shapes of the transverse vibrations of a shallow spherical shell with a completely free edge are given [13]:

$$\Phi_x^{(n)} = A_{pj} \{ (l^{p+4}/RD\mu_{pj}^4)C_{pj}\xi^p + J_p(\mu_{pj}, \xi) + D_{pj}I_p(\mu_{pj}, \xi)\}\cos p(\beta+\beta_0) \qquad (4-2)$$

where

p=the number of nodal diameters(meridians

j=the number of nodal circles

A_{pj}, C_{pj}, D_{pj} = the shape function coefficients

μ_{pj} = frequency parameter

D = bending stiffness factor

R = radius of curvature of the shell

J_p = the Bessel functions of the first kind

I_p = modified Bessel functions

Table 4–1 The first en natural frequencies of the shell

n	p	j	ω_n (rad./sec)
1	2	0	1.027715063
2	0	1	1.027781054
3	1	1	1.028163456
4	2	1	1.029176311
5	0	2	1.029465999
6	1	2	1.031988029
7	2	2	1.036220825
8	0	3	1.036928081
9	1	3	1.044596445
10	2	3	1.055618524

It is assumed that 12 point actuators are used to control attitude and shape of the shell system. The placement of the 12 actuators is determined by using the degree of controllability of the con rol system [13].The location of actuators on the shell are shown in Fig.4–1 and the ollowing Table 2.

Table 4–2 Locations of the 12 actuators on the shallow shell

Actuator No.	Locations of actuator (ξ,β) ξ	β	Directions of the jets f_x^0	f_y^0	f_z^0
1	0.84	0	1	0	0
2	0.84	π	1	0	0
3	0.57	$\pi/4$	1	0	0
4	0.57	$5\pi/4$	1	0	0
5	0.40	$3\pi/4$	1	0	0
6	0.40	$7\pi/4$	1	0	0
7	0.28	$\pi/2$	1	0	0
8	0.28	$3\pi/2$	1	0	0
9	1.00	$\pi/4$	1	$-\sin\pi/4$	$\cos\pi/4$
10	1.00	$5\pi/4$	1	$\sin\pi/4$	$-\cos\pi/4$
11	1.00	$3\pi/4$	1	$-\sin 3\pi/4$	$\cos 3\pi/4$
12	1.00	$7\pi/4$	1	$\sin 3\pi/4$	$-\cos 3\pi/4$

It is assumed that two Earth sensors and two Sun sensors are used to measure the attitude angle between the local vertical/the vector of the Sun direction and the roll axis, pitch axis of the shell, respectively. It is also assumed that 8 displacement sensors are used to measure shell's transverse displacement parallel to the shell's yaw axis; the displacement sensor are considered to becolocated with the actuators.

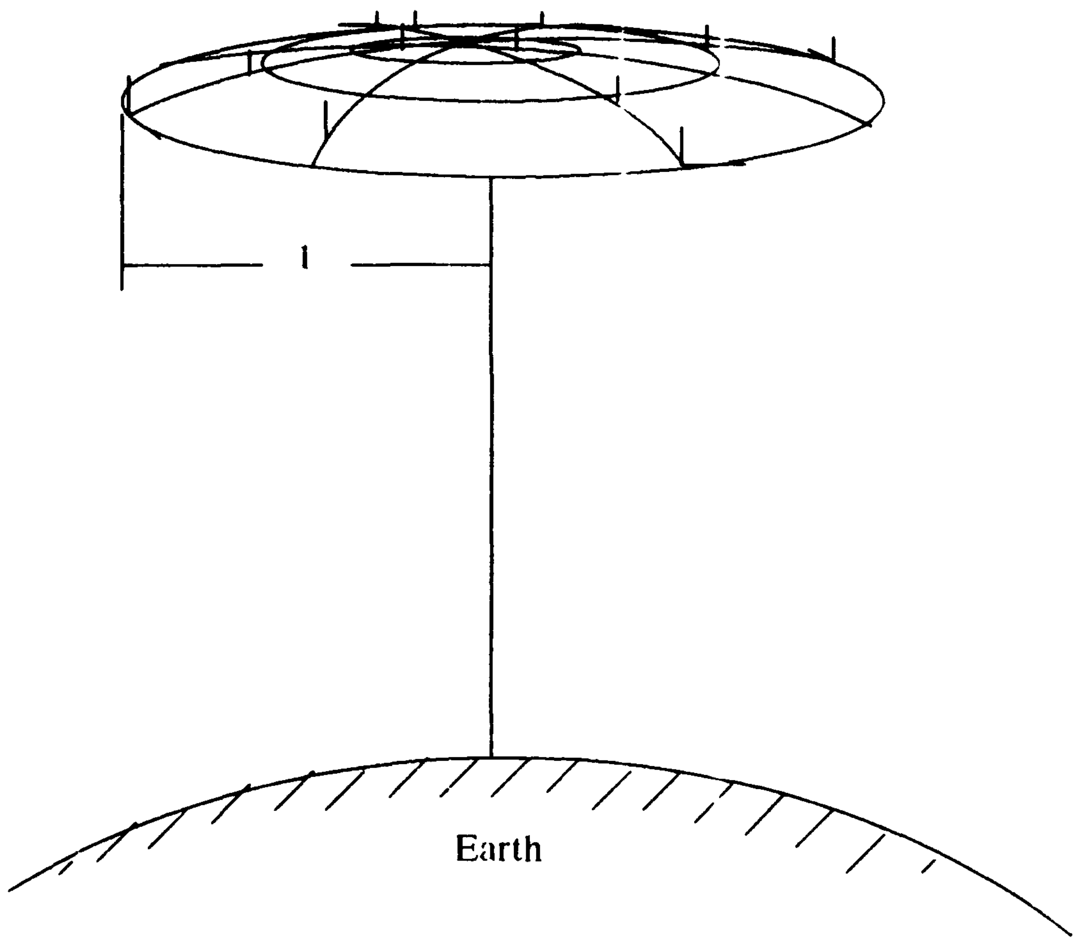


Fig. 4-1 Orbiting Shallow Spherical Shell System

The state equations of the system are as follows:

$$\dot{x} = A_c x + B_c u \tag{4-3}$$

where

$$x = (\psi, \dot{\psi}, \phi, \dot{\phi}, \theta, \dot{\theta}, \epsilon_1, \dot{\epsilon}_1, \epsilon_2, \dot{\epsilon}_2, \epsilon_3, \ldots, \epsilon_{10}, \dot{\epsilon}_{10})^T$$

Equations (3) can be discretized as follows:

$$X(k+1) = Ax(k) + Bu(k) \tag{4-4}$$

where

$$A = \exp(A_c T)\ , \qquad T = \text{sampling time}$$

$$B = \int_0^T \exp\{A_c(T-t)\}B_c dt$$

The discretized observation equation is as follows:

$$y(k) = Cx(k) \tag{4-5}$$

where

$$x(k) \in R^n, \qquad u(k) \in R^m, \qquad y \in R^r\ ;$$

$$A \in R^{n\times n}, \qquad B \in R^{n\times m}, \qquad C \in R^{r\times n}\ .$$

n=26, m=12, r=12 for the full order design model. It is assumed that the reduced order design model may be divided into 4 cases. They are shown in Table 4-3.

Table 4-3 Full order and reduced order design models

	The order of design model	Modes included in model	n	m	r
case 1	full order	rigid + 10 modes	26	12	12
case 2	18-dim reduced order	rigid + 6 modes	18	12	12
case 3	12-dim reduced order	rigid + 3 modes	12	12	12
case 4	8-dim reduced order	rigid + 1 modes	8	6	6
case 5	6-dim reduced order	rigid only	6	4	4

4.3 The Design of the Reduced Order LQG Controller and the Loop Transfer Recovery

4.3.1 Loop transfer recovery for the discrete-time system

It is assumed that the system state equation , measurement equation, control output equation and performance index for the LQG problem are as follows:

$$x(k+1) = Ax(k) + Bu(k) + L\xi(k) \tag{4-6}$$

$$y(k) = Cx(k) + \mu I\eta(k) \tag{4-7}$$

$$y_c(k) = Hx(k) \tag{4-8}$$

$$J = E\sum_{k=0}^{\infty} \{ x^T(k)\ \hat{Q}\ x(k) + u^T(k)\ \hat{R}\ u(k) \} \tag{4-9}$$

where

$$E\{(L\xi(k))(L\xi(k))^T\} = LL^T = Q \qquad E\{\xi(k)\xi^T(k)\} = I \tag{4-10}$$

$$E\ \{(\mu I\eta(k))(\mu I\eta(k))^T\} = \mu^2 I = R, \qquad E\{\eta(k)\eta(k)^T\} = I \tag{4-11}$$

$$\hat{Q} = H^T H, \qquad \hat{R} = \rho^2 I \tag{4-12}$$

It is well known that if the system (A,B,H) is controllable and observable, and the system (A,L,C) is controllable and observable (the conditions may be reduced to stabilizable and detectable for the time-invariant system), then the closed-loop system of the LQG optimal controller is asymptotically stable.

The LQG control law for the filter observer is :

$$u(k) = -K_c\hat{x}(k/k) \tag{4-13}$$

where

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K_f\ (y(k)-\hat{y}(k/k-1))$$
$$\hat{x}(k+1/k) = A\hat{x}(k/k-1) + Bu(k) + K_p\ (y(k) - \hat{y}(k/k-1))$$
$$\hat{y}(k/k-1) = C\hat{x}(k/k-1)$$
$$K_p = AK_f \qquad \text{filter gain matrix}$$
$$K_f = P_eC^T(CP_eC^T+R)^{-1}$$
$$P_e=AP_eA^T - AP_eC^T(R+CP_eC^T)^{-1}CP_eA^T+Q$$

and

$$K_c = (\hat{R} + B^TPB)^{-1}BPA \qquad \text{control gain matrix}$$

$$P = A^T P A - A^T P B(\hat{R}+B^T P B)^{-1} B^T P A + \hat{Q}$$

The diagram of the input and output in terms of the Z transformation is as follows:

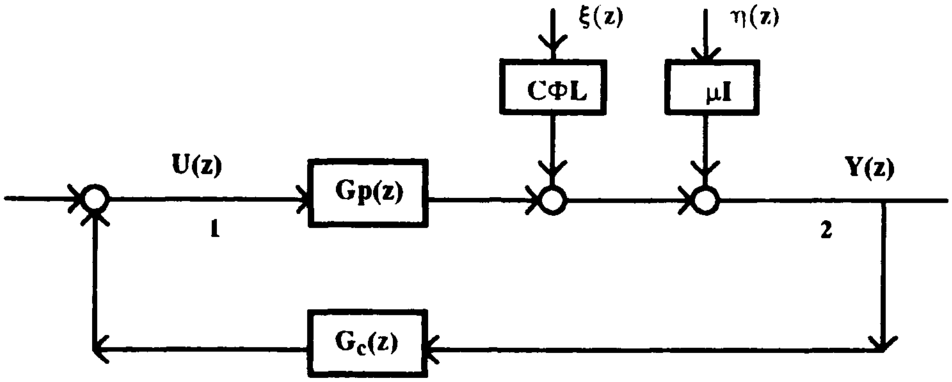


Fig4–2 Block diagram of the optimal LQG digital control with filtering observer

where

$$\Phi(z) = (zI-A)^{-1} \tag{4-14}$$

$$G_c(z) = zK_c[zI-(I-K_f)(A-BK_c)]^{-1}K_f \tag{4-15}$$

$$G_p(z) = C\Phi B \tag{4-16}$$

It is well known that the multivariable linear–quadratic optimal regulator(LQR) has impressive robustness properties. But if the observer or Kalman filter is used in the implementation, the robustness properties of the system will be degraded. The robustness recovery means that if the measurement noise parameter, μ, approaches zero, then the loop transfer function of the LQG control at the input loop–breaking point, 1, will approach the loop transfer function of the LQR control, i.e., robustness properties for the LQG control will be the same as for the LQR control when the noise parameter, μ, approaches zero.

In terms of mathematical notations, this property can be stated as follows:

If the open–loop transfer function of system (6)–(8) has no (finite)zeros in { z: |z|>1 } and det(CB)≠0, then

$$\lim_{\mu \to 0.} G_c G_p = K_c \Phi B \tag{4-17}$$

where the $K_c\Phi B$ is just the loop transfer function for the LQR control.

In the same way, the sensitivity recovery can be also proven, i. e.,

$$\lim_{\rho \to 0} G_pG_c = C\Phi K_p \tag{4-18}$$

where the $C\Phi K_p$ is just the loop transfer function for the Kalman filter.

It is evident that the robustness recovery properties for the LQG control may be used in the design of the reduced order controllers, so that the controllers designed based on the reduced order design models will have strong robustness properties.

4.3.2 The compromise between the performance and the robustness

It is well known that the basic requirements of a feedback system are:

(1) Stability: bounded output for all bounded disturbances and bounded reference input;

(2) Performance: small errors in the presence of disturbances and reference input;

(3) Robustness: stability and performance maintained in the presence of model uncertainties.

In fact, the requirements (1) and (2) are in conflict with the requirement of (3). In order to meet the requirement of (1) and (2), we should keep the sensitivity function[8]

$$S(z) = (I + G_p(z)G_c(z))^{-1} \tag{4-19}$$

as small as possible for all frequencies $z=\exp\{j\omega T\}$, ($-\omega_s/2 \leqslant \omega \leqslant \omega_s/2$, $T=2\pi/\omega_s$). If the requirement of (3) has to be met, we should keep the complementary sensitivity function $T(z)$,

$$T(z) = G_p(z)G_c(z)(I+G_p(z)G_c(z))^{-1} \tag{4-20}$$

as small as possible for all frequencies $z=\exp\{j\omega T\}$, ($-\omega_s/2 \leqslant \omega \leqslant \omega_s/2$, $T=2\pi/\omega_s$) However, since

$$S(z) + T(z) = I \tag{4-21}$$

they cannot be made small simultaneously. Rather we must trade off the size of one function against the size of the other in accordance with the relative importance of disturbance/command power and model uncertainty at each frequency.

Tradeoff between transfer functions can be formalized by posing them as function-space optimization problems[8]. The developments in [8] are for the continuous-time system; parallel to discussions in [8], the following developments can be obtained for the discrete-time systems.

The first thing needed for the optimization problem is to chose a convenient criterion of smallness. Consider

$$\sigma[M]^2 = \lambda_{max}[MM^H] < Tr[MM^H]$$

This shows that a matrix M will be small if $Tr[MM^H]$ is small. Using this latter measure for the two matrices, $S(e^{j\omega T})$ and $T(e^{j\omega T})$, adding weights $W(e^{j\omega T})$ to trade one against the other, and integrating over the frequency range results in the following optimization problem.

Given the plant $G_p(z)$, weight $W(z)$, and sensitivity and complementary sensitivity functions defined by (4−19) and (4−20), respectively, find a stabilizing compensator $G_c(z)$ to minimize

$$J = \int_{-\omega_s/2}^{\omega_s/2} \{Tr[SWW^HS^H + Tr[TT^H]\}d\omega = \int_{-\omega_s/2}^{\omega_s/2} Tr[MM^H]d\omega \qquad (4-22)$$

where

$$M=M(e^{j\omega T}) = [S(e^{j\omega T})W(e^{j\omega T}),\ T(e^{j\omega T})]$$

In mathematical terms, this represents an H^2−optimization problem[17]. Using this approach similar to that in [8], it can be proven that the LQG problem for the discrete-time system defined by (4−6)−(4−12) may be converted into the equivalent H^2−optimization problem, i.e., the performance index, (4−9), can be converted into the following formulation:

$$J_{lqg} = (T/2\pi)\int_{-\omega_s/2}^{\omega_s/2} Tr[P(e^{j\omega T})P^H(e^{j\omega T})]d\omega \qquad (T=2\pi/\omega_s) \qquad (4-23)$$

where

$$P(e^{j\omega T}) = \begin{pmatrix} H\Phi L-H\Phi BG_c(I+G_pG_c)^{-1}C\Phi L & -\mu H\Phi BG_c(I+G_pG_c)^{-1} \\ & \\ -\rho G_c(I+G_pG_c)^{-1}C\Phi L & -\rho\mu G_c(I+G_pG_c)^{-1} \end{pmatrix} \qquad (4-24)$$

Comparing (4−22) and (4−23), it is now apparent that the only remaining step needed to solve (4−22) is to find free parameters for (4−24) such that P(z) reduces to M(z). It is easy to verify that the following choices provide the desired result.

Choose L and μ such that (Sensitivity Recovery)

$$C\Phi L/\mu = W(z) \tag{4-25a}$$

and let

$$H{=}C \text{ and } \rho \rightarrow 0 \tag{4-25b}$$

then

$$P(z) \longrightarrow \mu \begin{pmatrix} (I+G_pG_c)^{-1}W & -G_pG_c(I+G_pG_c)^{-1} \\ 0 & 0 \end{pmatrix} \tag{4-25c}$$

$$= \mu \begin{pmatrix} SW & -T \\ 0 & 0 \end{pmatrix}$$

Note that for each nonzero value of ρ, the LQG solution for these choices produces an appropriate stabilizing controller which is H^2–optimal for (4–23). Moreover, since (4–23) converges to (4–22), a sequence of decreasing ρ-values produces a sequence of controllers which optimizes (4–22) in the limit.

It is also easy to verify that the following alternative to (4–25) produces another useful transfer function tradeoff:

Choose H and ρ such that (Robustness Recovery)

$$H\Phi B/\rho = W(z) \tag{4-26a}$$

and let

$$L{=}B \text{ and } \mu \rightarrow 0 \tag{4-26b}$$

Then

$$P(z) \longrightarrow \rho \begin{pmatrix} W(I+G_cG_p)^{-1} & 0 \\ -(I+G_cG_p)^{-1}G_cG_p & 0 \end{pmatrix} \tag{4-26c}$$

$$= \rho \begin{pmatrix} WS & 0 \\ -T & 0 \end{pmatrix}$$

These choices accomplish an H^2–tradeoff between the sensitivity and complementary sensitivity functions at the input loop–breaking point 1 of Fig.4–1 instead of at the output. Both choices (4–25) and (4–26), will be referred to as LQG/LTR.

Eq.(4–17) (or (4–18)) shows that the optimal loop transfer function matrix of a minimum–phase H^2–problem (4–22) corresponds to the loop transfer function of the LQR(or Kalman filter)problem with its loop breaking point 1(or 2). Moreover, Eq.(4–17) (or (4–18)) shows that the sequence of LQG solutions generated by (4–26)(or (4–25)) converges to this function("recovers" this function) as the design parameter μ(or ρ) becomes small. These results may be listed as follows:

Table 4–4 Robustness (Sensitivity) Recovery

Robustness Recovery	Sensitivity Recovery
Let $L = B$, $r \geqslant m$ Suppose $C(zI-A)^{-1}L$ is min. phase When $\mu \rightarrow 0$ then $T_1(z) \rightarrow K_c(zI-A)^{-1}B$	Let $H = C$, $r \leqslant m$ Suppose $H(zI-A)^{-1}B$ is min. phase When $\rho \rightarrow 0$ then $T_2(z) \rightarrow C(zI-A)^{-1}K_f$
Relationship between LQ regulator parameter and sensitivity weighting $H(zI-A)^{-1}B/\rho = W(z)$ A, B, H, ρ : LQ – regulator parameters $W(z)$: Sensitivity weighting	Relationship between Kalman filter parameter and sensitivity weighting $C(zI-A)^{-1}L/\mu = W(z)$ A, L, C, μ : Kalman filter parameters $W(z)$: Sensitivity weighting

Where

$X \in R^n$, $U \in R^m$, $Y \in R^r$
$T_1(z)=G_c(z)G_p(z)$: The transfer function at the input loop–breaking point 1
$T_2(z)=G_p(z)G_c(z)$: The transfer function at the output loop–breaking point 2
$K_c(zI-A)^{-1}B$: LQR loop transfer function
$C(zI-A)^{-1}K_f$: Kalman filter loop transfer function

The significance of these result is that we can design LQG loop transfer functions on a full–state feedback basis and then approximate them adequately with a recovery procedure. For the point 1, the full–state design must be done with the LQR equations and recovery with the Kalman filter, while for point 2, full–state design must be done with Kalman filter equations and recovery with LQR equations.

The above properties suggest a two–step approach to the H^2–optimal problem design:

Step 1: Design a LQ–regulator (or Kalman filter)via (4–26a)(or (4–25a)), with desirable sensitivity, complementary sensitivity, and loop transfer function.

Step 2: Design a sequence of a Kalman filter(or LQ–regulators), via (4–26b) (or(4–25b)), to approximate the function in step 1 to whatever robustness(or accuracy) is needed.

Both of these steps are easy design tasks The first is easy because the LQ–regulator(or Kalman filter), sensitivity, complementary sensitivity, and loop transfer function are explicity related to the chosen weights $W(e^{j\omega T})$, and the second is easy because it involves only repeated solutions of the algebraic Riccati equations. This follows by inspection of S and T.

4.3.3 The design of reduced order optimal digital LQG controllers

It is our expectation that the reduced order controllers should maintain as much robustness as possible when the performance of the system satisfies the design requirements. Therfore, a two–step robustness recovery procedure will be applied as follows:

(1) Design LQ–regulators via (4–26a) for the full order system For all frequencies where the weights $W(e^{j\omega T})=H\Phi(e^{j\omega T})B/\rho$ are much larger than unity, the LQ–regulator sensitivity, complementary sensitivity, and loop transfer function have the following properties.

$$\sigma_i\left[(I+K_c\Phi(e^{j\omega T})B)^{-1}\right] = 1/\sigma_i\left[W(e^{j\omega T})\right] \qquad (4-27)$$

$$\sigma_i\left[K_c\Phi(e^{j\omega T})B(I+K_c\Phi(e^{j\omega T})B)^{-1}\right] = 1 \qquad (4-28)$$

$$\sigma_i\left[K_c\Phi(e^{j\omega T})B\right] = \sigma_i\left[W(e^{j\omega T})\right] \qquad (4-29)$$

for each singular value, σ_i. Therefore, the accuracy may be improved by properly increasing the weight, $W(z)$, The adjustible parameters are H and ρ for this case; if H has been selected, the accuracy may be improved by proper reduction of the parameter, ρ. The preliminary design of the LQ–regulator just involces the selection of the parameter, ρ, in this case.

(2) Design a sequence of Kalman filters via (4–26b) for the full order system Let L=B, and $\mu \longrightarrow 0$, to approximate the function in step 1 to whatever robustness is needed. The preliminary design of the Kalman filter involves just the selection of the parameter, μ.

(3) Simulations of LQG optimal digital control for various reduced order controllers for the selected parameter pair (ρ,μ). The simulations will be used for verifying which of the available reduced order controllers is best.

4.4 Numerical Results

It is assumed that the plant model of the orbiting shallow spherical shell includes 3 rigid modes, 3 axisymmetric modes, 1 meridional mode, 6 combined modes, i.e., 26 dimensions in all. The controllers may be divided into 5 cases:

Case 1 : full order controllers
Case 2 : reduced order 18–dim controllers
Case 3 : reduced order 12–dim controllers
Case 4 : reduced order 8–dim controllers
Case 5 : reduced order 6–dim controllers

The parameters for the simulations are selected as follows:
Sampling time: 5 seconds
System noise : $\sigma_s = 10^{-3}$ (rad.)
Observational noise : $\sigma_o = 10^{-2}$ (meter)

Physical and geometrical parameters of the shell:
mass = 10,000 kg.
the base radius of shell = 100 meters
the height of shell = 1 meter
radius of curvature for shell = 5000 meters
wall thickness of shell = 0.01 meter

Initial conditions for all simulations:
$\psi(0)=\phi(0)=\theta(0)=0.2$ rad.
$\dot{\psi}(0)=\dot{\phi}(0)=\dot{\theta}(0)=0.02$ rad./sec
$q_1(0)=q_2(0)=q_3(0)=\ldots=q_6(0)=5$ meters
$q_7(0)=q_8(0)=\ldots=q_{10}(0)=0$
$\dot{q}_1(0)=\dot{q}_2(0)=\ldots=\dot{q}_{10}(0)=0$

(1) The design of the desirable LQ regulators

The free parameters for the regulators are H and ρ. If we let H=C, the ρ becomes the only free parameter. The weighting function and Eq.(27) indicate that with ρ properly reduced, the loop's errors in the presence of commands and disturbances can be made small. But ρ cannot be too small; otherwise the errors in the presence of observational noise will increase with the reduction of ρ, and the robustness will be degraded, often resulting in divergence of the transient responses for the reduced order controllers. Fig.4–3 certifies this point and shows that $\rho=1$ is a proper value.

(2) The design of the desirable Kalman filter

By means of the procedures for the robustness recovery, as we know, the robustness of the LQG control system willbe increased when μ is reduced; but the μ cannot be too small, otherwise the accuracy of the LQG control system will be degraded. The design of the control system involves a compromise between robustness and accuracy. Fig.4–4 certifies this point. The transient responses show that the proper μ values are 0.1 or 0.01

(3) The determination of the reduced order LQG controllers in the presence of unmodeled dynamic uncertainties.

Simulations have been conducted for the LQG control (plant is 26 dimensions) with 4 kinds of reduced order controllers(18–dim, 12–dim, 8–dim.and 6–dim) for the orbiting shallow spherical shell, where the parameters, ρ and μ are set as 1 and 0.1, respectively. Fig.4–5 shows that the robustness and performance of the reduced order LQG controllers are worse than that of the full order controllers; the 12–dim reduced order controller is sufficient for handling the unmodeled dynamics of the shallow spherical shell system; the 6–dim and 8–dim reduced order controllers cannot satisfy the requirement of the optimal control of the shallow shell system; the 6–dim controller, which is based only on the rigid modes, will result in severe divergence for the transient responses of the shallow shell system.

4.5 Summary

(1) The properties of robustness recovery(sensitivit recovery) for discrete–time systems are studied, and may be used for the design of educed order optimal LQG digital controllers forthe shallow spherical shell system. The design of the control system, in fact, is a compromise between robustness and performance of the control system after the stability of the control system is satisfied. If L=B and H=C, the robustness and performance of the control system only depend on two parameters: ρ(sensitivity) and μ(robustnesst). Therefore, the design of the LQG control system involves the selection of the proper parameter pair (ρ,μ), so that the system is stable and the robustness and performance satisfy the design requirements.

(2) If the robustness recovery is used for increasing the robustness of the control system, in general, the μ value should be as small as possible; but considering the performance of the system, the μ cannot be selected too small, otherwise the performance of the system will be degraded. In general, the performance of the reduced order controllers is worse than that of the full order controllers.

(3) Simulations have certified the 12–dim reduced order controller will be sufficient for the optimal LQG control of the shallow spherical shell system in the presence of unmodeled dynamics. The performance of the 8–dim reduced order LQG controllers for the shell system is unacceptable. The 6–dim reduced order controllers for the shell system will result in the severe divergence of the transient responses.

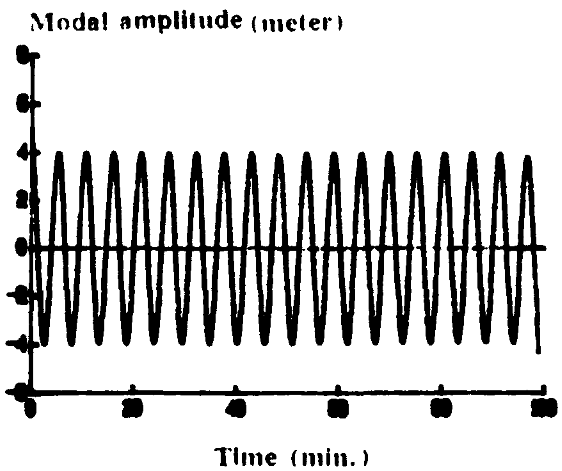


Fig.4-3-1 ρ=100 (full order)

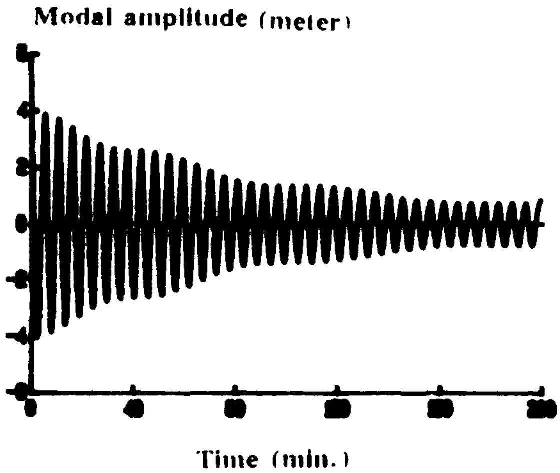


Fig.4-3-2 ρ=1 (full order)

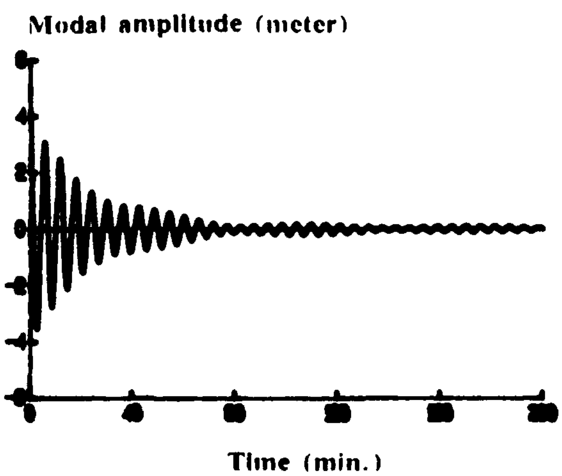


Fig.4-3-3 ρ=0.1 (full order)

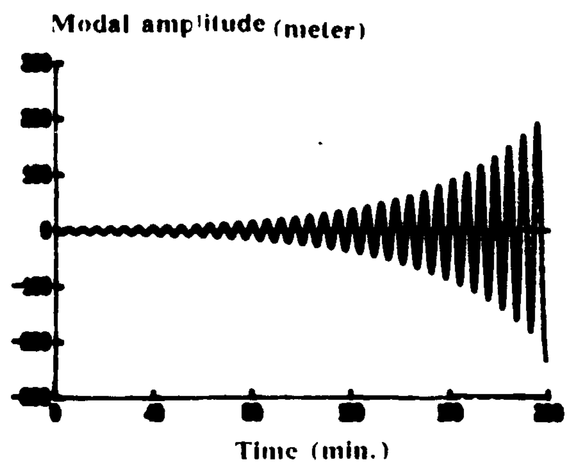


Fig.4-3-4 ρ=0.1 (12-dim. reduced order)

Fig.4-3 The Influence of ρ in the Controller on the Transient Response of LQG Control for an Orbiting Shallow Spherical Shell System

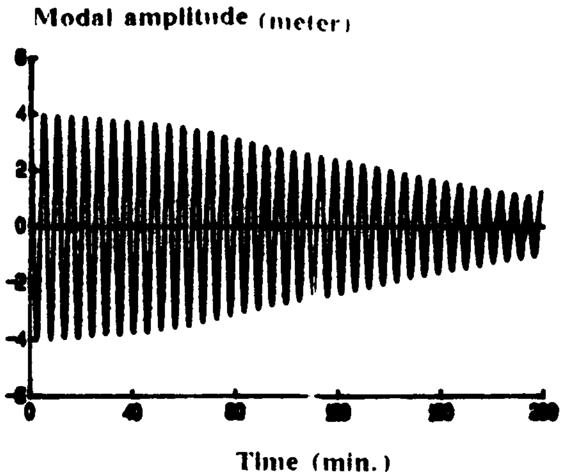


Fig.4-4-1 μ=1 (full order)

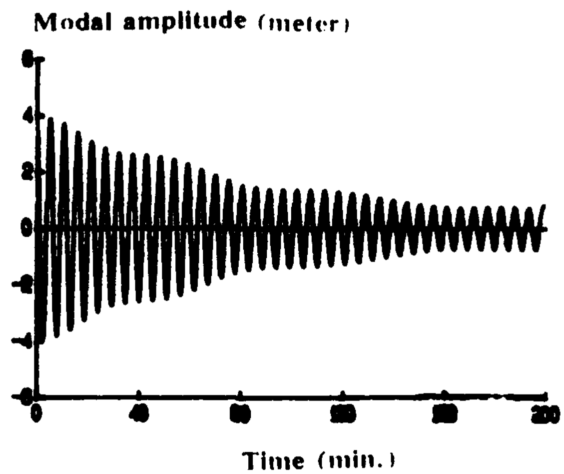


Fig.4-4-2 μ=0 (full order)

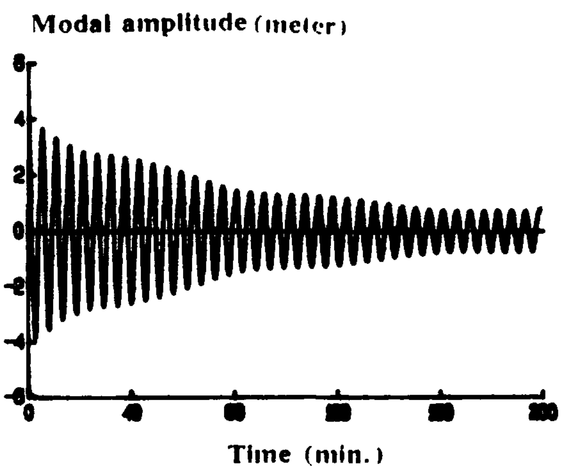


Fig.4-4-3 μ=0.01 (full order)

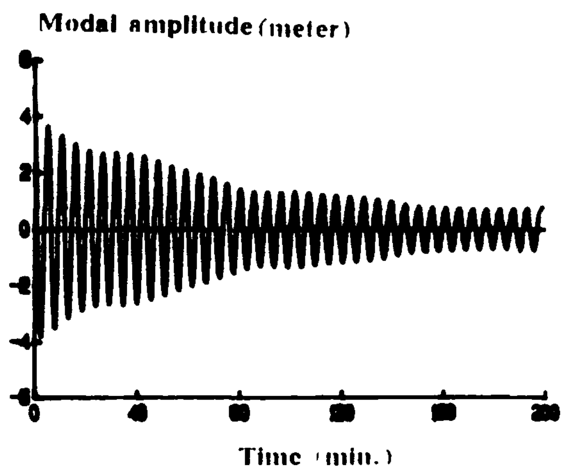


Fig.4-4-4 μ=0.001 (full order)

Fig.4-4 The Influence of μ in the Controller on the Transient Response of LQG Control for an Orbiting Shallow Spherical Shell System

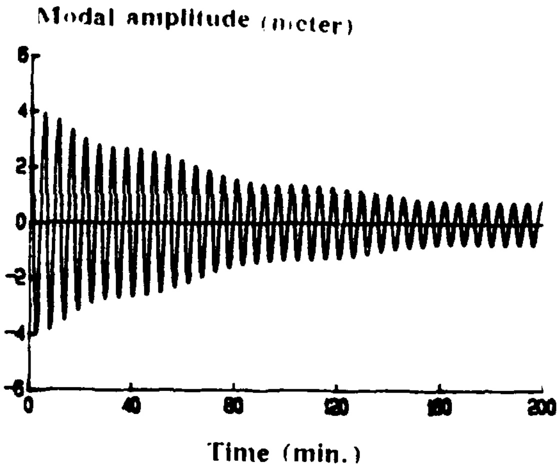


Fig.4-5-1 18-dim reduced order controller

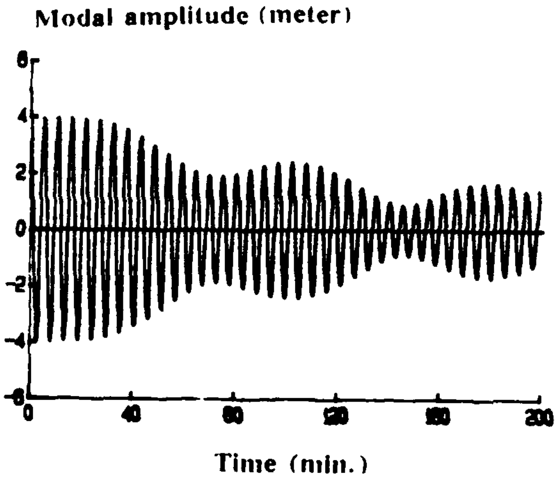


Fig.4-5-2 12-dim reduced order controller

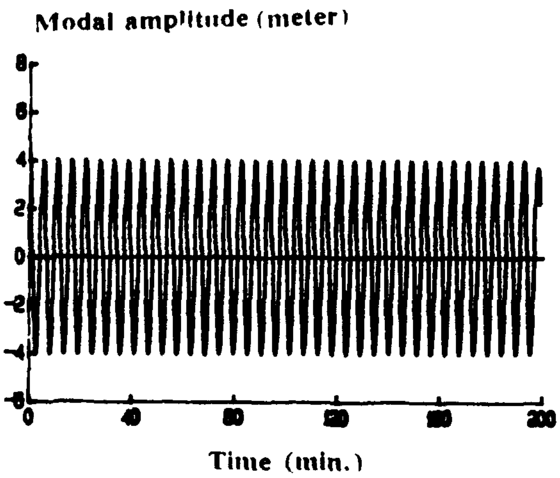


Fig.4-5-3 8-dim reduced order controller

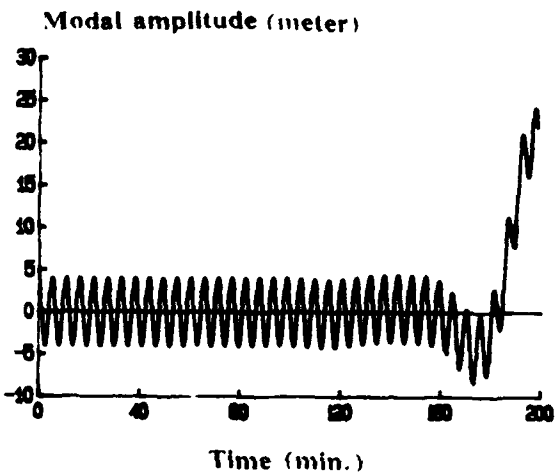


Fig.4-5-4 6-dim reduced order controller

Fig.4-5 The Robustness Comparison of Various Reduced Order LQG Controllers for an Orbiting Shallow Spherical Shell System (ρ=1, μ=0.1)

5 The Comparison Between the Digital Optimal LQG Controller with the Filtering observer and Predicting Observer for the Orbiting Flexible Shallow Spherical Shell System

As we know, there are two kinds of different LQG controllers for the discrete-time system: one of them is shown Fig.3-1, in which the state being fed into the controller is the filtered estimate of the state variable, so it called the digital LQG controller with filtering observer; another is shown Fig.3-2, in which the state being fed into the controller is the predicted estimate of the state variable, so it is called the digital LQG controller with the predicting observer.

It was shown in Chapter 3 that there is a robustness recovery property for the digital LQG controller with filtering observer, but there is no corresponding property for the digital LQG controller with the predicting observer. The proof is given in detail in Chapter 3. In this chapter, the comparison between the transient responses of the LQG digital control with filtering observer and predicting observer will be made by simulations of the LQG digital control for the orbiting shallow spherical shell system(Fig.4-1).

It is assumed that the plant model of the orbiting shallow spherical shell includes 3 rigid modes, 3 axisymmetric modes, 1 meridional mode, 6 combined modes, i.e., 26 dimensions in all. The controllers may be divided into 3 cases:

Case 1 : full order controllers (3 rigid modes + 10 flexible modes)
Case 2 : 18-dim reduced order controllers(3 rigid modes + 6 flexible modes)
Case 3 : 12-dim reduced order controllers(3 rigid modes + 3 flexible modes)

The physical and geometrical parameters of the shell:
mass = 10,000 kg.
the base radius of shell = 100 meters
the height of shell = 1 meter
the radius of curvature for shell = 5000 meters
wall thickness of shell = 0.01 meter

The parameters for the simulations are selected as follows:
sampling time: 5 seconds
system noies: $\sigma_s = 10^{-3}$ (rad.)
observational noise: $\sigma_o = 10^{-2}$ (meter)

Initial conditions for all simulations:
$\psi(0)=\phi(0)=\theta(0)=0.2$ rad.
$\dot{\psi}(0)=\dot{\phi}(0)=\dot{\theta}(0)=0.02$ rad./sec.
$q_1(0)=q_2(0)=q_3(0)=\ldots=q_6(0)$ = 5 meters
$q_7(0)=q_8(0)=\ldots=q_{10}(0)=0.0$
$\dot{q}_1(0)=\dot{q}_2(0)=\ldots=\dot{q}_{10}(0)$

Based on the development in the Chapter 4, the relationship between the robustness recovery and the accuracy of the control system may be listed as follows:

Table 5-1 Robustness recovery and system accuracy

Robustness Recovery	System Accuracy
Let $L = B$, $r \geqslant m$ Suppose $C\Phi L$ is min. phase where $\Phi = (zI-A)^{-1}$	Sensitivity function $S(z) = (1+T_1(z))^{-1}$ Sensitivity weighting $W(z) = H\Phi B/\rho$
When $\mu \rightarrow 0$ then $T_1(z) \rightarrow K_c\Phi B$	when $\mu \rightarrow 0$ then $\sigma_i(S(z)) \rightarrow \sigma_i((1+K_c\Phi B)^{-1}) = 1/\sigma_i(W)$

where

$T_1(z)=G_c(z)G_p(z)$: the transfer function of the input loop-breaking point 1
$K_c\Phi B$: LQR loop transfer function

In the design of LQG robust control systems, the free parameters are, ρ, the control weighting factor in performance index and, μ, the measurement noise parameter. As shown in Table 5-1, the reduction of μ will increase the system robustness; the reduction of ρ will reduce the $\sigma_i(S(z))$. $S(z)$ is the sensitivity function of the system, the reduction of $\sigma_i(S)$ means that the accuracy of the control system will be improved.

5.1 Comparison of transient responses between the full order LQG digital controller with predicting observer and filtering observer

In order to study and compare the influence of the sensitivity parameter, ρ, and robustness parameter, μ, on the transient response of the digital full order LQG controller with predicting observer and filtering observer, the parameters ρ, and μ are varied in the combination shown in Table 5-2 and Table 5-3 for the simulations:

Table 5-2 The parameter pair (ρ,μ) in Fig. 5-1

Case	ρ	μ	Predicting observer	Filtering observer
1-1	1	1	Fig.5-1-1	Fig.5-1-2
1-2	1	0.1	Fig.5-1-3	Fig.5-1-4
1-3	1	0.01	Fig.5-1-5	Fig.5-1-6

Table 5-3 The parameter pair (ρ,μ) in Fig. 5-2

Case	ρ	μ	Predicting observer	Filtering observer
1-4	0.1	1	Fig.5-2-1	Fig.5-2-2
1-5	0.1	0.1	Fig.5-2-3	Fig.5-2-4
1-6	0.1	0.01	Fig.5-2-5	Fig.5-2-6

The following two points are shown in Fig.5-1 and Fig.5-2:

1 Fig.5-1 shows that the transient response of the digital full order LQG controller with filtering observer for the orbiting shallow spherical shell system is more sensitive to the reduction of the parameter, μ, than that of the digital full order LQG controller with the predicting observer. The transient performance of the system with filtering observer is better than that of the system with predicting observer for the same parameter pair (ρ,μ).

2 Fig.5-2 shows that the decay of the transient response of the system will increase strongly with the reduction of the sensitivity parameter, ρ; i.e., the time of the transient process of system will be shortened when the value of ρ is reduced. In this case the phenomenon of the data saturatim will also appear earlier in the response, but the accuracy of the system with filtering observer is still better than the system with predicting observer.

5.2 Comparison of transient responses between the 18-dim reduced order LQG digital controller with predicting observer and filtering observer

In order to study and compare the influence of the sensitivity parameter, ρ, and robustness parameter, μ, on the transient response of the digital 18-dim reduced order LQG controller with predicting observer and filtering observer, the parameters ρ, and μ are varied in the combinations shown in the Tables 5-4 and 5-5 for the simulations:

Table 5-4 The parameter pair (ρ,μ) in Fig. 5-3

Case	ρ	μ	Predicting observer	Filtering observer
2-1	1	1	Fig.5-3-1	Fig.5-3-2
2-2	1	0.1	Fig.5-3-3	Fig.5-3-4
2-3	1	0.01	Fig.5-3-5	Fig.5-3-6

Table 5-5 The parameter pair (ρ,μ) in Fig. 5-4

Case	ρ	μ	Predicting observer	Filtering observer
2-4	0.1	1	Fig.5-4-1	Fig.5-4-2
2-5	0.1	0.1	Fig.5-4-3	Fig.5-4-4
2-6	0.1	0.01	Fig.5-4-5	Fig.5-4-6

The following points are shown in Fig.5-3, and Fig.5-4:

1 Fig.5-3 shows that the 18-dim. reduced order LQG digital controller for the both types of observer are stable for the parameters ρ=1 and μ=1, 0.1, 0.01; The transient response of the system will be improved when the value of the parameter, μ, is decreased from 1 to 0.01. The performance of the system with the filtering observer is better than that of the system with the predicting observer when the value of the parameter, μ, is reduced.

2 Fig.5-4 shows that the transient response of the system with the filtering observer for the orbiting shallow spherical shell system is still stable when the values of the parameter, μ, is reduced, but the transient response of the system with the predicting observer is not stable for the parameters ρ=0.1, μ=0.01. This result is due to the fact that the sensitivity of the system will be increased when the value of the parameters, ρ, is decreased from 1 to 0.1. This indicates that the robustness of the system is reduced with the reduction of the parameter, ρ. Therefore, the transient response of the system with predicting observer will result in divergence since the robustness of system is not sufficient to cover the error of the unmodelled dynamics; but the transient response of the system with the filtering observer is still stable due to the robustness recovery property of the system with filtering observer.

5.3 Comparison of transient responses between the 12-dim reduced order LQG digital controller with predicting observer and filtering observer

In order to study and compare the influence of the sensitivity parameter, ρ, and the robustness parameter, μ, on the transient response of the digital 12-dim. reduced order LQG controller with the predicting observer and filtering observer, the parameters,

ρ, and μ are combined indicated in Tables 5–6 and 5–7 for the simulations:

Table 5–6 The parameter pair (ρ,μ) in Fig. 5–5

Case	ρ	μ	Predicting observer	Filtering observer
3–1	1	1	Fig.5–5–1	Fig.5–5–2
3–2	1	0.1	Fig.5–5–3	Fig.5–5–4
3–3	1	0.01	Fig.5–5–5	Fig.5–5–6

Table 5–7 The parameter pair (ρ,μ) in Fig. 5–6

Case	ρ	μ	Predicting observer	Filtering observer
3–4	0.1	1	Fig.5–6–1	Fig.5–6–2
3–5	0.1	0.1	Fig.5–6–3	Fig.5–6–4
3–6	0.1	0.01	Fig.5–6–5	Fig.5–6–6

The following points are shown in the Fig 5–5 and Fig. 5–6:

1 Fig. 5–5 shows that the 12–dim. reduced order LQG digital controller for both observers are stable for $\rho = 1$, and μ=1,0.1,0.01. The transient response of the system will be improved when the value of μ decreases from 1 to 0.01. The performance of the system with the filtering observer is better than that of the system with the predicting observer when the value of μ is reduced.

2 Fig. 5–5 and Fig. 5–6 show that transient response of the systems with predicting observer and filtering observer are not stable. Because the system robustness is reduced when the value of ρ is reduced from 1 to 0.01, and the error of unmodelled dynamics for the 12–dim. reduced order controller is greater than that for the 18–dim. reduced order controller, the robustness of these systems is not sufficient to overcome the unmodelled dynamics. Therefore, the divergence of the system's transient response results for all parameter pairs (ρ,μ) in Fig. 5–6.

What we should point out is that the divergence of the system with the filtering observer is slower than that of the system with the predicting observer.

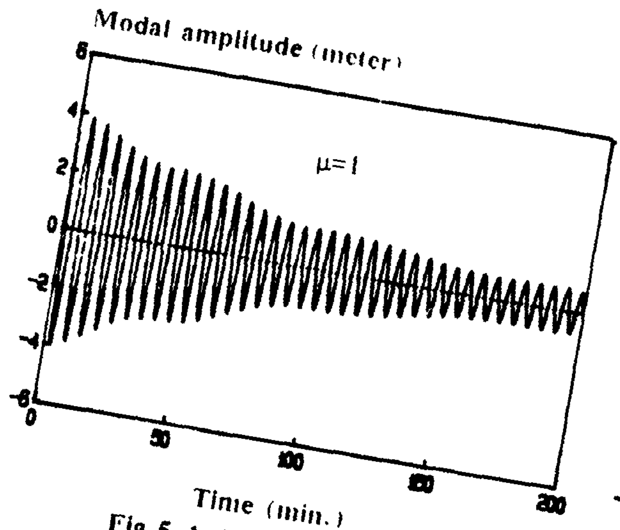


Fig.5-1-1 Predicting observer

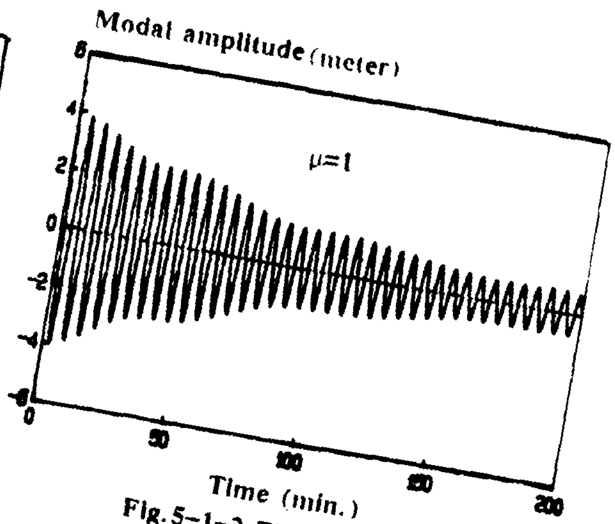


Fig.5-1-2 Filtering observer

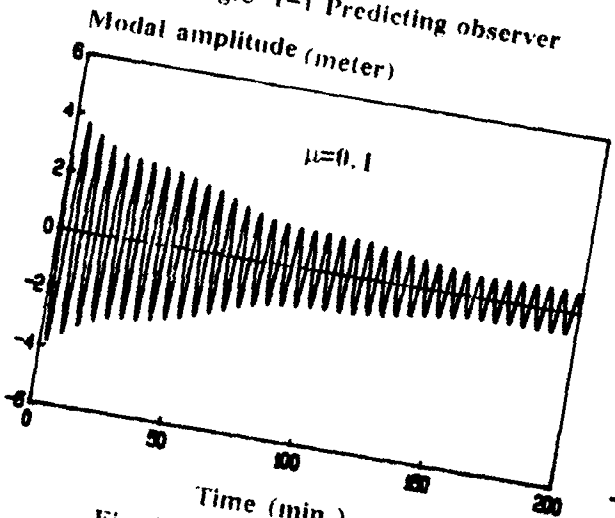


Fig. 5-1-3 Predicting observer

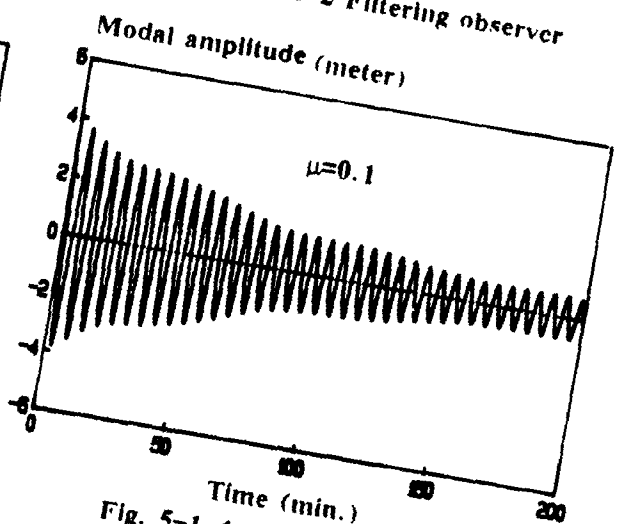


Fig. 5-1-4 Filtering observer

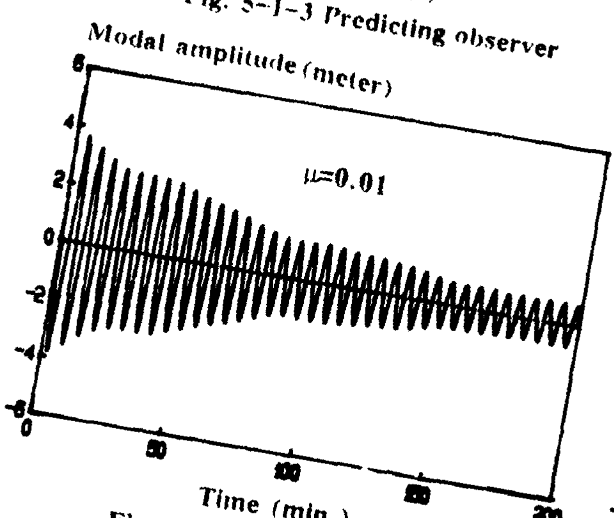


Fig.5-1-5 Predicting observer

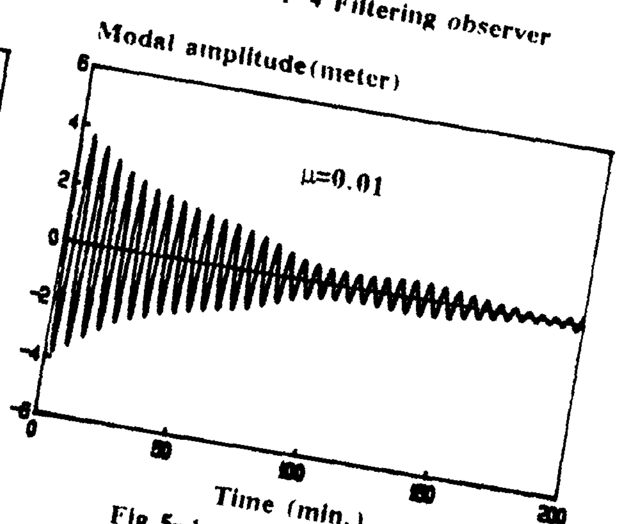


Fig.5-1-6 Filtering observer

Fig.5-1 The Comparison Between the Modal Amplitude Response of the Full Order LQG Controller with Predicting Observer and Filtering Observer (ρ=1)

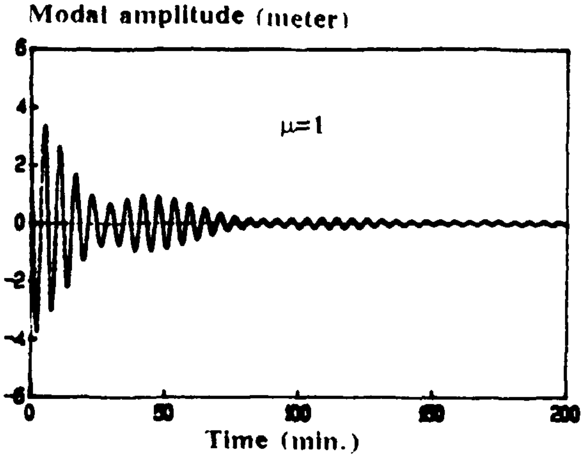


Fig.5-2-1 Predicting observer

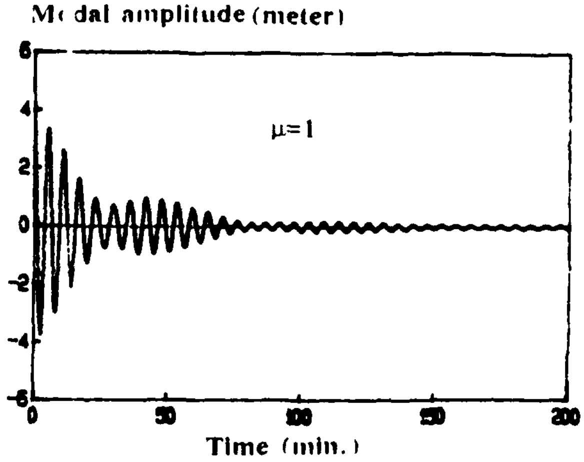


Fig.5-2-2 Filtering observer

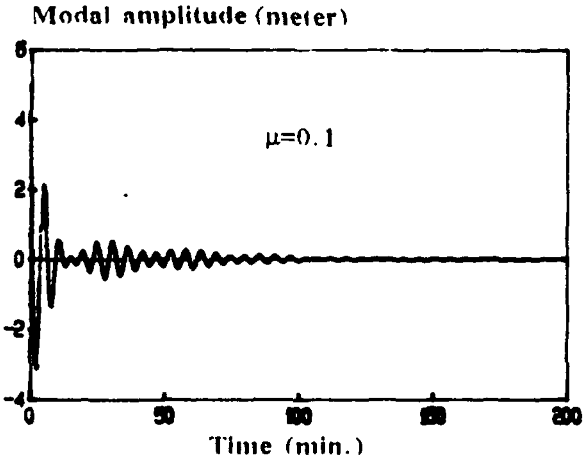


Fig. 5-2-3 Predicting observer

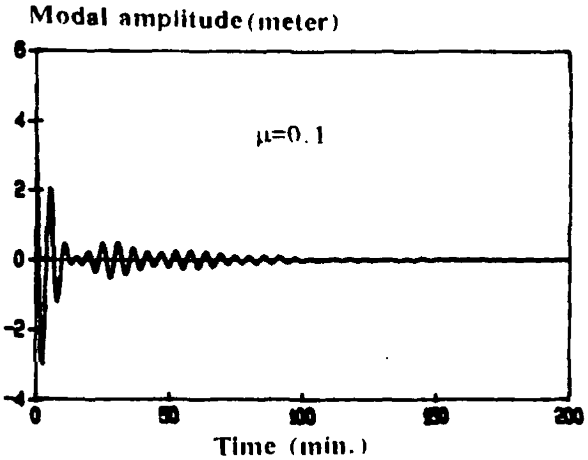


Fig. 5-2-4 Filtering observer

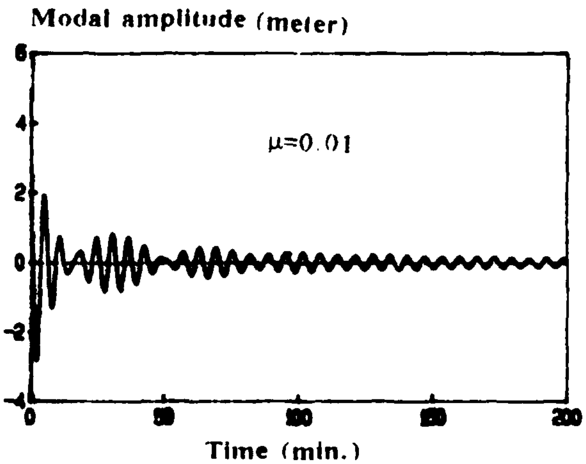


Fig.5-2-5 Predicting observer

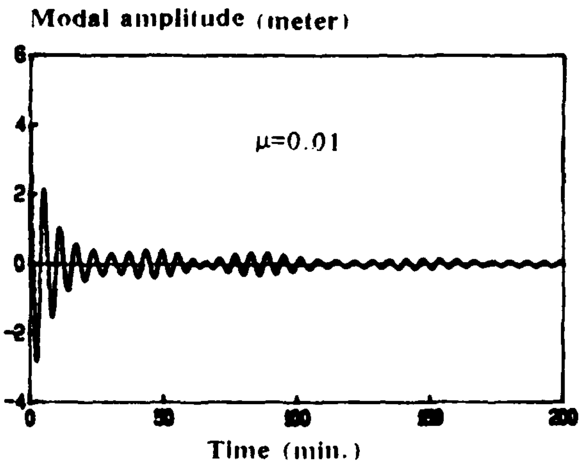


Fig.5-2-6 Filtering observer

Fig.5-2 The Comparison Between the Modal Amplitude Response of the Full Order LQG Controller with Predicting Observer and Filtering Observer (ρ=0.1)

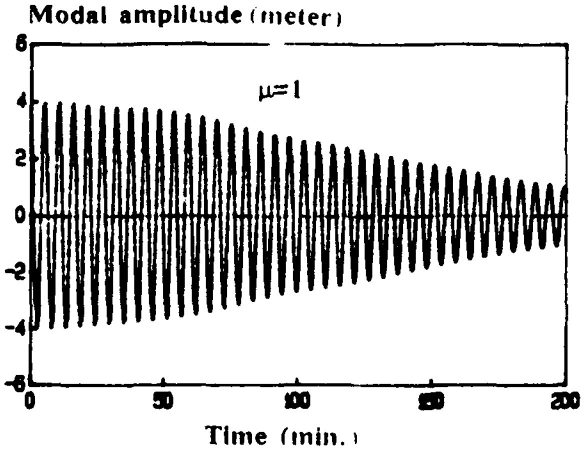


Fig.5-3-1 Predicting observer

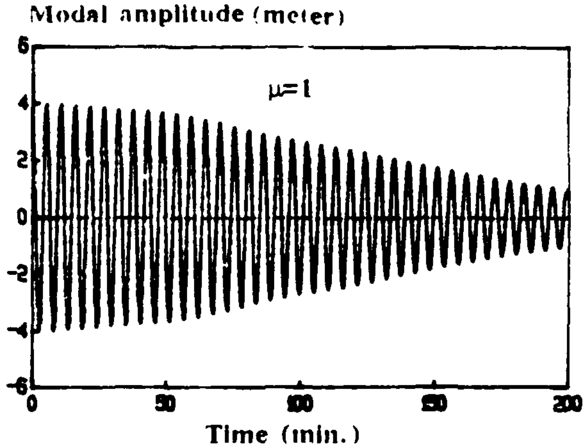


Fig.5-3-2 Filtering observer

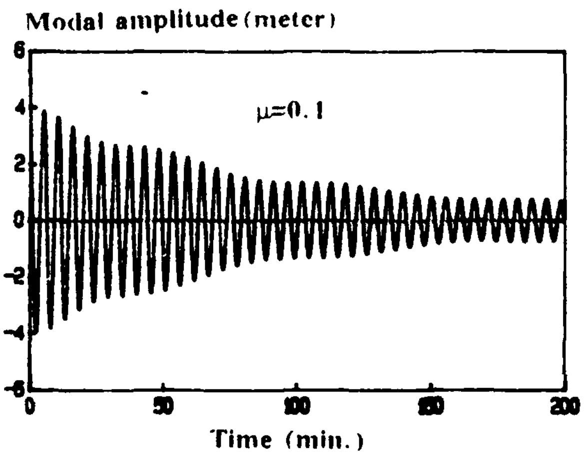


Fig. 5-3-3 Predicting observer

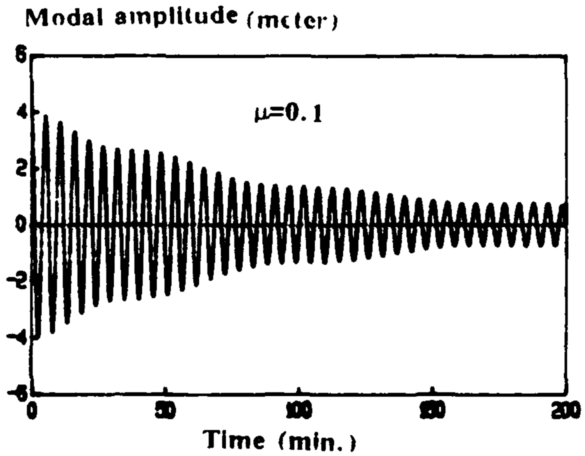


Fig. 5-3-4 Filtering observer

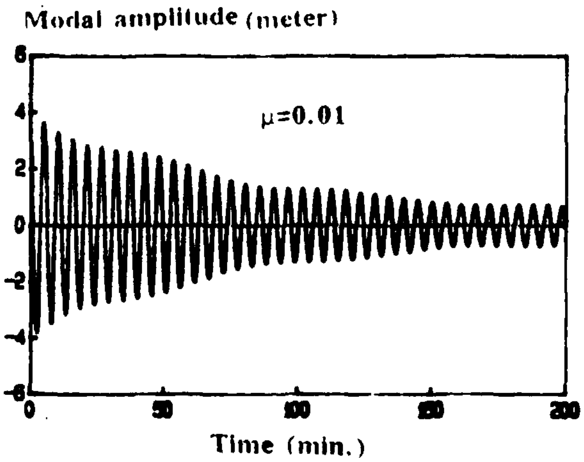


Fig.5-3-5 Predicting observer

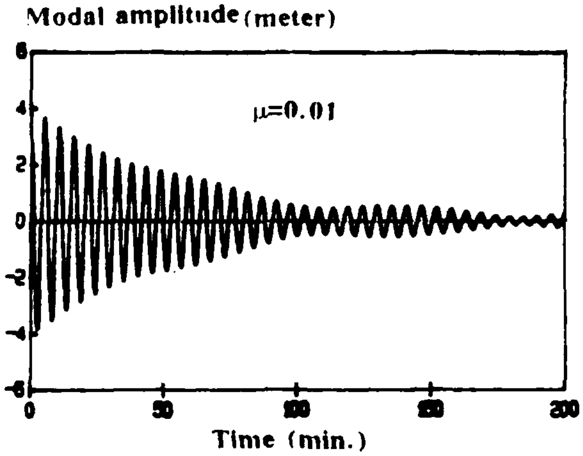


Fig.5-3-6 Filtering observer

Fig.5-3 The Comparison Between the Modal Amplitude Response of the 18-Dim. Reduced Order LQG Controller with Predicting Observer and Filtering Observer (ρ=1)

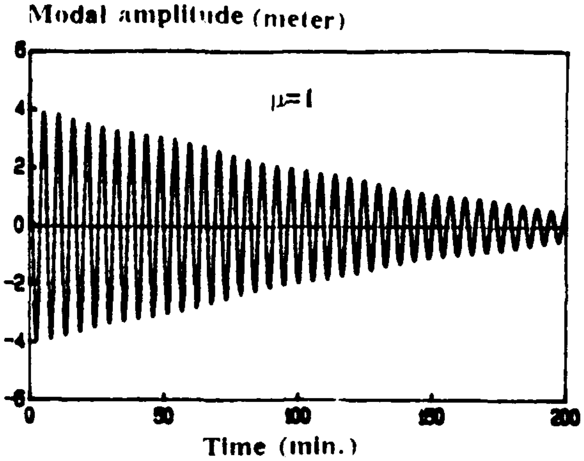


Fig.5-4-1 Predicting observer

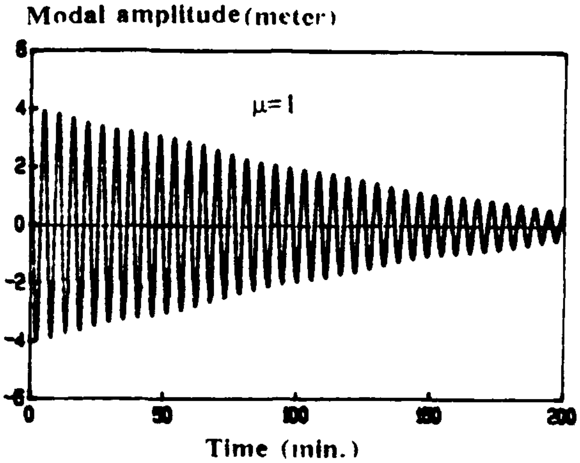


Fig.5-4-2 Filtering observer

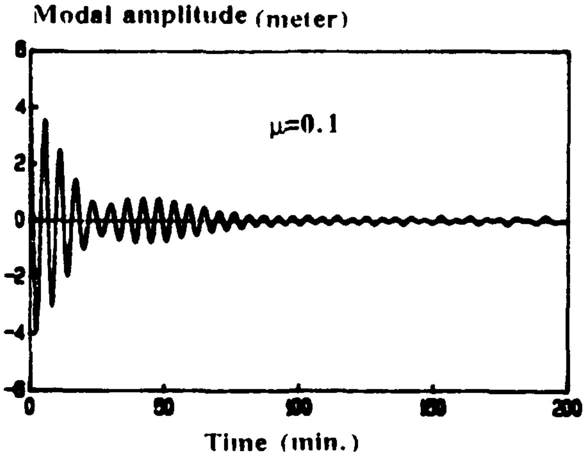


Fig. 5-4-3 Predicting observer

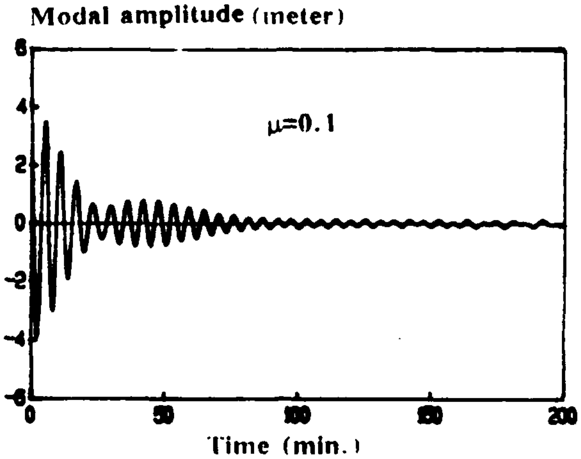


Fig. 5-4-4 Filtering observer

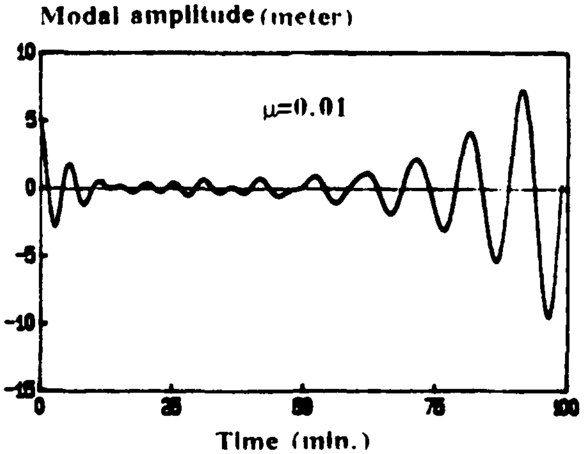


Fig.5-4-5 Predicting observer

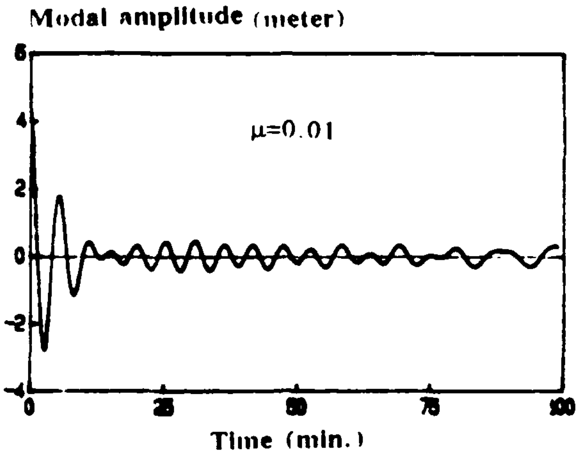


Fig.5-4-6 Filtering observer

Fig.5-4 The Comparison Between the Modal Amplitude Response of the 18-Dim. Reduced Order LQG Controller with Predicting Observer and Filtering Observer (ρ=0.1)

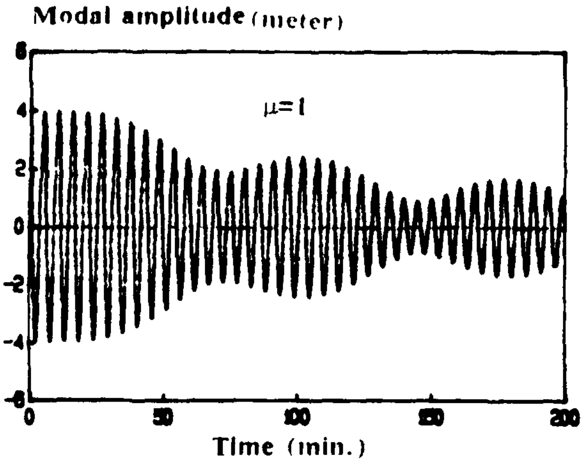


Fig.5-5-1 Predicting observer

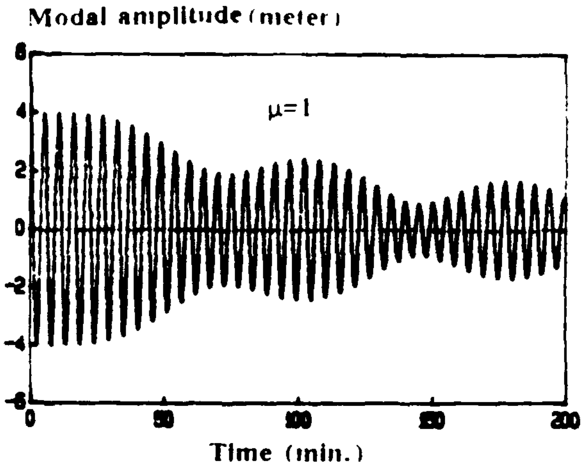


Fig.5-5-2 Filtering observer

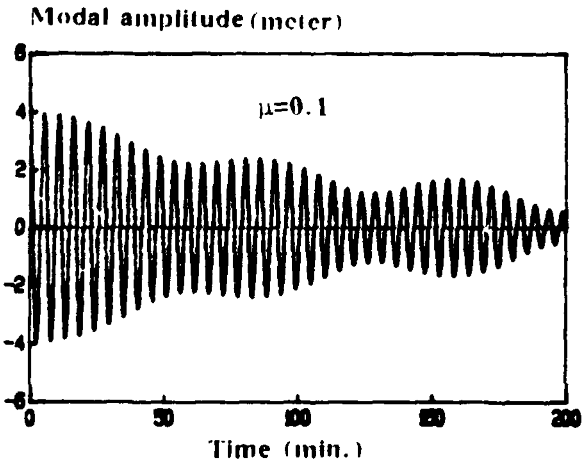


Fig. 5-5-3 Predicting observer

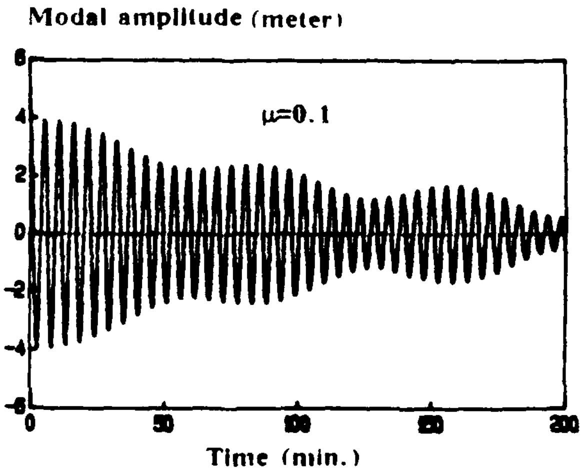


Fig. 5-5-4 Filtering observer

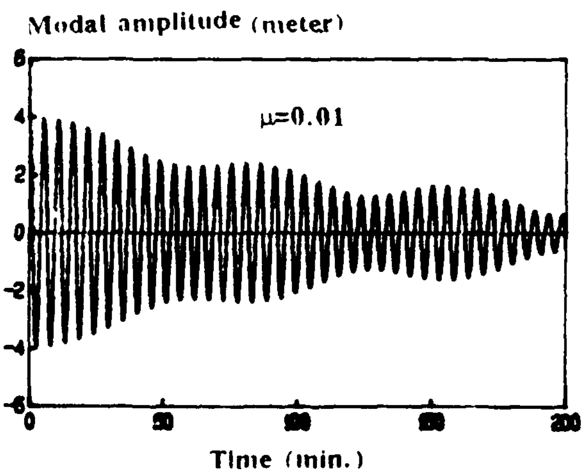


Fig.5-5-5 Predicting observer

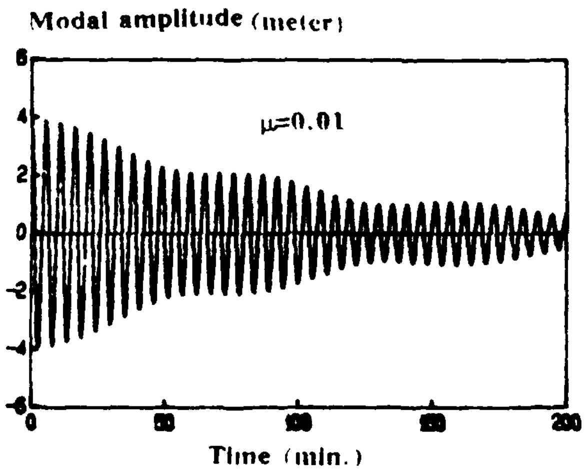


Fig.5-5-6 Filtering observer

Fig.5-5 The Comparison Between the Modal Amplitude Response of the 12-Dim Reduced Order LQG Controller with Predicting Observer and Filtering Observer (ρ=1)

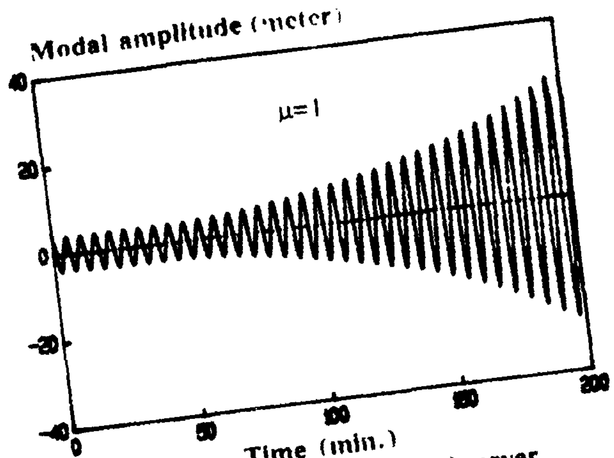


Fig.5-6-1 Predicting observer

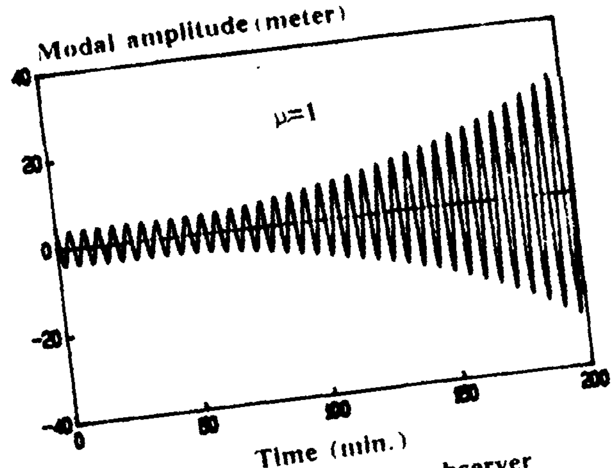


Fig.5-6-2 Filtering observer

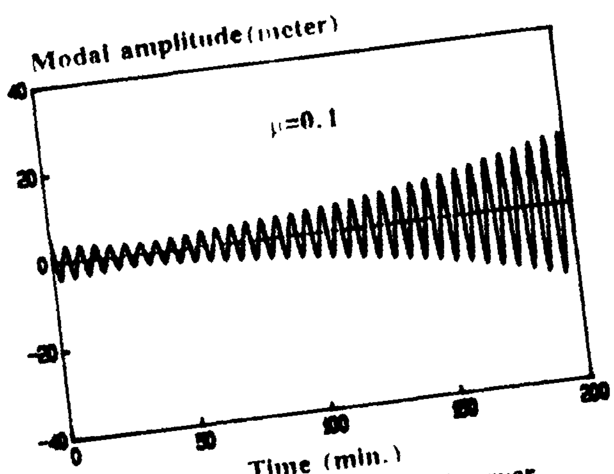


Fig. 5-6-3 Predicting observer

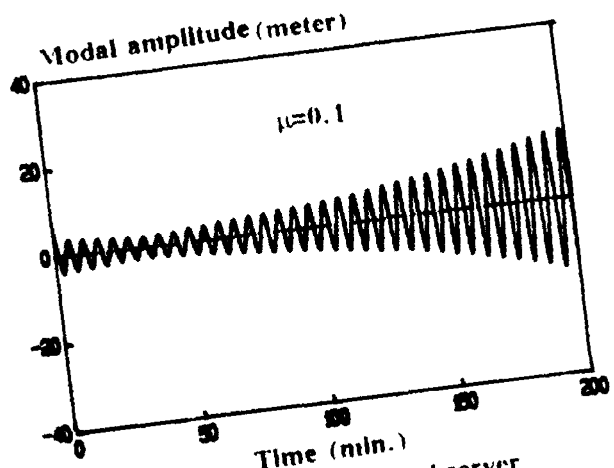


Fig. 5-6-4 Filtering observer

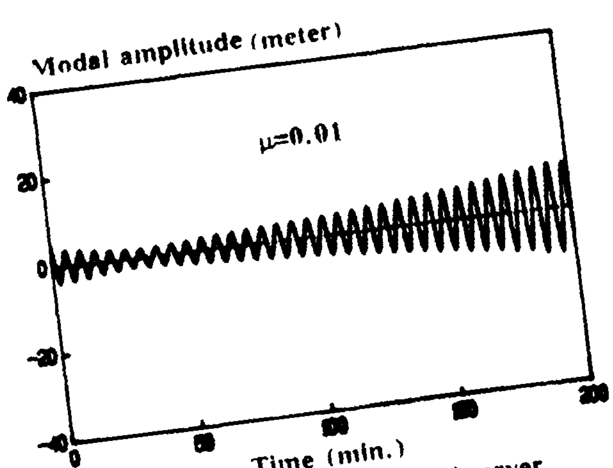


Fig.5-6-5 Predicting observer

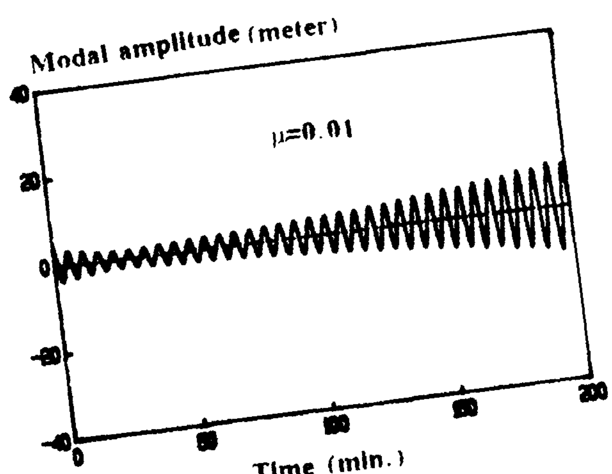


Fig.5-6-6 Filtering observer

Fig.5-6 The Comparison Between the Modal Amplitude Response of the 12-Dim. Reduced Order LQG Controller with Predicting Observer and Filtering Observer (ρ=0.1)

6 General Conclusions and Suggested Future Direction

6.1 General conclusions

In order to develop the analysis and design methods for robustness control of large space structure sampled data stochastic systems with a specific application to the orbiting flexible shallow spherical system, the following problems have been studied and the conclusions we have obtained are as follows:

(1) The theory of multi–input, multi–output(MIMO) transfer function matrices in the z transformation may be used for the analysis and design in the frequency domain for the discrete–time system. The frequency response of the transfer function matrix for the discrete–time system can be obtained by means of the frequency response of the transfer function matrix for the equivalent continous–time system.

(2) The robustness criteria in the frequency domain for discrete–time systems have been developed. The stability conditions for MIMO discrete–time feedback system with additive alteration are (2–9) or (2–10); The stability conditions for MIMO discrete–time feedback with multiplicative alteration are (2–14) or (2–16).

(3) One of the most important breakthroughs in multi–input, multi–output feedback system theory for the last decade is the development of the loop transfer recovery methodology for the continous–time linear quadractic Gaussian problem, which is called LQG/LTR. The LQG/LTR technique has been extended from the continous–time system to the discrete–time system in this report. It is proven that the robustness (sensitivity) recovery property is also valid for the LQG digital controller with the filtering observer, but it is not valid for the LQG digital controller with the predicting observer.

(4) As an application of the LQG/LTR technique for discrete–time systems to large space structural systems, the LQG/LTR technique of the discrete–time systems has been used to design the reduced order optimal digital LQG controller for the orbiting flexible shallow spherical shell system. The research results indicate that the 12–dim reduced order controller will be sufficient for the optimal LQG control of the shallow spherical shell system in the presence of unmodelled dynamics.

(5) The comparisons between the digital optimal LQG controller with the filtering observer and predicting observer for the orbiting flexible shallow spherical shell system have been made. The robustness recovery property for the digital LQG controller with filtering observer has been certified by the simulations. The simulations indicate that the transient response of the digital LQG control system with the filtering observer or with the predicting observer, in general, depends on the robustness parameter, μ, and sensitivity parameter, ρ. The best combination of the parameters ρ and μ, will depend on the compromise between the accuracy and robustness. Considering the robustness recovery property, the system performance of the robust control system with the filtering observer will be better than that of the LQG robust control system with the predicting observer.

6.2 The suggested future directions

(1) Loop transfer recovery for nonminimum phase plants for discrete–time systems

The requirement of minimum phase plant (i.e., the transfer function of the plant has no (finite) zero outside the unit circle) for the recovery procedure is critical. Since there are some plants which are minimum phase systems in practice, it would be desirable to have a methodology for incorporating limitations due to non–minimum phase zeros into the LTR procedure. It is especially more desirable for discrete–time systems, since the standard sampling process is known to introduce zeros, some of which sometime lie outside the unit circle.

(2) Synthesis and design of reduced order LQG/LTR optimal digital controllers using constrained optimization techniques

It is well known that the basic requirements of a feedback system are better performance(small error in the presence of disturbances and reference input) and robustness(stability and performance maintained in the presence of model uncertainties). In fact, the two parts of these basic requirements are in conflict with each other. As far as the LQG/LTR method is concerned, the conflict is reflected in the selections of the robustness parameter and sensitivity parameter[8]. Therefore, we may convert the problem of reduced order LQG/LTR controller design into the constrained optimization problem. This procedure minimizes a linear quadratic Gaussian(LQG) type cost function while trying to satisfy a set of constraints on the responses and stability margins. Although a linear LQG cost function was minimized by updating the free parameters of the control law, while satisfying a set of constraints on the design loads, responses, and stability margin[18], our attention will be focused on the design and synthesis of the reduced order LQG/LTR optimal digital controller for discrete–time systems, using only a small number of design parameters specifically associated with robustness and sensitivity. As an application, this method will be applied to the design of digital reduced order LQG/LTR controllers for the orbiting shallow spherical shell system.

(3) The synthesis and design of the robust digital optimal output feedback reduced order controllers using constrained optimization

Since the mathematical system model is inherently of high order for large space structural systems and because of the practical possibility of on–board computational implementation, it is desirable to have methods avaible for the design of low–order controllers for high–order plants. Such methods can be broadly divided into two classes: (a) direct method: in which the parameters defining a low–order controller are computed by some optimization or other procedure; (b) indirect method: in which a high–order controller is first found and then a procedure used to simplify, or a low–order plant first is found by some criterion, and then a low–order controller based on the simplified low–order plant is designed. In general, the direct method is better than the indirect method in meeting the requirement of the designer. The design method for the robust digital optimal output feedback reduced order controller using constrained optimization is just the direct design method for the low–order controller. Therefore, it is very useful to study and develop the design method for the robust digital optimal output feedback reduced order controller using constrained optimization.

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